Junior Questions: Part A

Each question should be answered by a single choice from A to E. Questions are worth 3 points each.

1. Garden Plots  
A vegetable garden is laid out in hexagonal plots. In the diagram below, the shaded plots are growing tomatoes. Marigolds will be planted in some of the adjacent plots to deter pests. The number in each shaded plot is the number of adjacent plots that will be planted with marigolds.

![Diagram of hexagonal garden plots]

Which number should be in plot A?

(A) 0  (B) 1  (C) 2  (D) 3  (E) 4

2. SMS  
In order to reduce the length of your SMS messages, you use the following rules:

• remove all spaces;
• remove all vowels (a, e, i, o, u);
• replace double letters with single letters;

For example, the message good morning would be sent as gdmrng, saving 6 characters (1 space, 4 vowels, 1 double letter).

You send the following three messages:

two white asters
five violet tulips
three red daisies

How many characters do you save in total?

(A) 23  (B) 25  (C) 27  (D) 29  (E) 31
3. Cities 1

The land of Straightopia has four cities, all built along a single straight highway. The distances between the cities are as follows:

<table>
<thead>
<tr>
<th>Distance</th>
<th>City P</th>
<th>City Q</th>
<th>City R</th>
<th>City S</th>
</tr>
</thead>
<tbody>
<tr>
<td>City P</td>
<td>3 km</td>
<td>3 km</td>
<td>1 km</td>
<td></td>
</tr>
<tr>
<td>City Q</td>
<td>3 km</td>
<td>6 km</td>
<td>4 km</td>
<td></td>
</tr>
<tr>
<td>City R</td>
<td>3 km</td>
<td>6 km</td>
<td>2 km</td>
<td></td>
</tr>
<tr>
<td>City S</td>
<td>1 km</td>
<td>4 km</td>
<td>2 km</td>
<td></td>
</tr>
</tbody>
</table>

You are travelling along the highway from one end to the other. In which order might you travel past the four cities?

(A) Q, P, S, R  (B) Q, R, P, S  (C) R, P, S, Q
(D) S, R, P, Q  (E) S, R, Q, P
4. Robot

The weather is lovely, so — like any other sensible person — you decide to spend a day on the beach with your pet robot.

Your robot is fairly simple; all it can do is walk around the beach and trace patterns in the sand. It begins facing north and accepts the following instructions:

- \( F_x \): Walk forwards \( x \) centimetres.
- \( L_x \): Stay in the same place and turn \( x \) degrees to the left.
- \( R^n \): Repeat whatever appears in the brackets \( n \) times.

For example, the robot can trace out an equilateral triangle in the sand using the instructions \( F30\ L120\ F30\ L120\ F30\ L120 \) (see the illustration below). This same pattern can be simplified by writing \( R3\ [\ F30\ L120\ ] \), since the forward/left instructions are repeated three times.

![Equilateral Triangle](image)

Note that if you repeat the forward/left instructions four times instead of three, the pattern in the sand would be exactly the same (the robot simply walks over one edge of the triangle twice).

You are feeling artistic, and you would like the robot to trace out the pattern illustrated below.

![Pattern](image)

You have given the robot the following instructions:

\[
R2\ [\ R4\ [\ F50\ L90\ ]\ L90\ ]
\]

Unfortunately it does not trace out the pattern that you wanted. With a slap of the forehead, you realise that the number in one of your instructions was wrong. Which instruction had the incorrect number?

(A) \( R2 \)  (B) \( R4 \)  (C) \( F50 \)  (D) The first \( L90 \)  (E) The second \( L90 \)
Junior Questions: Part B

Each question should be answered by a number in the range 0–999. Questions are worth 2 points each.

5–7. Borrowing Albums 1

A library has several different albums of a band. They are in a stack in order 1 2 3 \ldots N, with 1 on top. When an album is borrowed, the order of the remaining albums does not change. For instance, if N was 5 and album 3 was borrowed, the stack would become 1 2 4 5.

Several albums are borrowed, and a week later returned. As they are returned they are put on top of the stack in the order that they are returned.

For each of the following questions, the order shown is the order in the stack after the albums have been returned.

What is the smallest number of albums that has been borrowed?

5. 4 3 2 1

6. 3 6 2 5 1 4

7. 2 4 6 8 1 3 5 7 9

8–10. Rescue

It is a dark and stormy night, and you are watching over the troubled seas off the coast of Tasmania. You are leading a team of three rescue craft, stationed in the water and ready for any emergency.

The sea can be pictured as a grid of regions, such as the grid illustrated below. Your three rescue craft are stationed in three of these regions. Although they are swift and sturdy, they can only travel north, south, east and west through the grid (in particular, they cannot travel along diagonals). A rescue craft can move through one square in one minute.

Each region of the sea is protected by the rescue craft closest to it, as measured by the time it takes each craft to reach it. If a region is equally close to two rescue craft, it is protected by both of them. The diagram below shows the regions protected by each craft in the grid above.
Your task is to identify which rescue craft is protecting the most regions. In this case it is the second craft, which protects eight regions.

Each of the following scenarios describes the positioning of the three rescue craft within the sea. For each scenario, what is the largest number of regions protected by a single rescue craft? As an example, your answer for the scenario illustrated above would be 8.
1. Stars 2005 I.1

Beginning with some number \( n \), you write a line of ‘*’s by repeatedly applying the following rules:

- If \( n \) is 0, stop.
- If \( n \) is odd, write a single ‘*’ and reduce \( n \) by 1.
- If \( n \) is even, divide \( n \) by 2.

For example, if you begin with \( n = 3 \) then you would proceed as follows. Since 3 is odd, you write a single ‘*’ and subtract one to give \( n = 2 \). Since 2 is even, you divide by two giving \( n = 1 \). Finally, since 1 is odd you write another ‘*’ and subtract one. Now \( n = 0 \) and you stop, having written two ‘*’s in total.

If you begin with the number \( n = 77 \), how many ‘*’s do you write in total?

(A) 2  (B) 3  (C) 4  (D) 5  (E) 6

2. Buried Treasure 2010 I.2

The treasure has been buried, but the pirates do not trust each other. They agree that each of them will have part of the instructions needed to locate the treasure. They all know where to start, so the instructions are of the form “\( m \) metres E (or W), \( n \) metres N (or S)”.

After several hours of calculation, they produce the following set of instructions:

- Olaf: 100 metres E, 60 metres N
- Bluebeard: 100 metres E, 60 metres S
- Hook: 100 metres W, 60 metres N
- Sinbad: 100 metres W, 40 metres N
- Noah: 100 metres E, 40 metres N

Alas, despite all of their sums, they got it wrong. The location of the buried treasure is 200 metres E and 80 metres N. It can be found using 4 of the instructions, but not all 5.

Whose instructions should not be in the set?

(A) Olaf  (B) Bluebeard  (C) Hook  (D) Sinbad  (E) Noah
3. Cities 2

The land of Pitopia is centred upon a large circular lake. Around this lake is a circular highway, with five cities placed along the highway. The distances between the cities are as follows:

<table>
<thead>
<tr>
<th>Distance</th>
<th>City P</th>
<th>City Q</th>
<th>City R</th>
<th>City S</th>
<th>City T</th>
</tr>
</thead>
<tbody>
<tr>
<td>City P</td>
<td>5 km</td>
<td>3 km</td>
<td>6 km</td>
<td>4 km</td>
<td></td>
</tr>
<tr>
<td>City Q</td>
<td>5 km</td>
<td>2 km</td>
<td>1 km</td>
<td>3 km</td>
<td></td>
</tr>
<tr>
<td>City R</td>
<td>3 km</td>
<td>2 km</td>
<td>3 km</td>
<td>5 km</td>
<td></td>
</tr>
<tr>
<td>City S</td>
<td>6 km</td>
<td>1 km</td>
<td>3 km</td>
<td>2 km</td>
<td></td>
</tr>
<tr>
<td>City T</td>
<td>4 km</td>
<td>3 km</td>
<td>5 km</td>
<td>2 km</td>
<td></td>
</tr>
</tbody>
</table>

Note that there are always two different ways of travelling from one city to another (corresponding to the two different directions around the lake); the table above lists the shorter distance in each case.

You are travelling along the highway in a constant direction around the lake. In which order might you travel past the five cities?

(A) P, Q, S, T, R  (B) P, R, S, T, Q  (C) P, R, Q, T, S  
(D) P, S, Q, T, R  (E) P, T, S, Q, R
4. Robot

The weather is lovely, so — like any other sensible person — you decide to spend a day on the beach with your pet robot.

Your robot is fairly simple; all it can do is walk around the beach and trace patterns in the sand. It begins facing north and accepts the following instructions:

- \( F_x \) : Walk forwards \( x \) centimetres.
- \( L_x \) : Stay in the same place and turn \( x \) degrees to the left.
- \( R^n[...] \) : Repeat whatever appears in the brackets \( n \) times.

For example, the robot can trace out an equilateral triangle in the sand using the instructions \( F_{30} \ L_{120} \ F_{30} \ L_{120} \ F_{30} \ L_{120} \) (see the illustration below). This same pattern can be simplified by writing \( R_3 \ [ \ F_{30} \ L_{120} \ ] \), since the forward/left instructions are repeated three times.

\[
\triangle
\]

Note that if you repeat the forward/left instructions four times instead of three, the pattern in the sand would be exactly the same (the robot simply walks over one edge of the triangle twice).

You are feeling artistic, and you would like the robot to trace out the pattern illustrated below.

\[
\square
\]

You have given the robot the following instructions:

\[
R_3 \ [ \ R_3 \ [ \ F_{50} \ L_{120} \ ] \ L_{60} \ ]
\]

Unfortunately it does not trace out the pattern that you wanted. With a slap of the forehead, you realise that the number in one of your instructions was wrong. Which instruction had the incorrect number?

(A) The first \( R_3 \)  (B) The second \( R_3 \)  (C) \( F_{50} \)  (D) \( L_{120} \)  (E) \( L_{60} \)
5–7. Borrowing Albums 2

A library has several different albums of a band. They are in a stack in order 1 2 3 ... N, with 1 on top. When an album is borrowed, the order of the remaining albums does not change. For instance if N was 5 and album 3 was borrowed, the stack would become 1 2 4 5.

Several albums are borrowed, and a week later returned. When they are returned they are put anywhere in the stack. In the example above, if album 3 was returned the stack could be 3 1 2 4, 1 3 2 4, 1 2 3 4, or 1 2 4 3.

For each of the following questions, the order shown is the order in the stack after the albums have been returned.

What is the smallest number of albums that could have been borrowed?

5. 4 2 5 6 1 3 7

6. 4 2 3 6 5 7 8 1

7. 2 4 3 1 5 8 6 7 9
8–10. Ports

An island is divided into several regions, as illustrated on the grid below. A few of these regions are ports; these are marked with shaded squares. Each of the remaining regions charges a small tax for travellers who pass through the region; these taxes are indicated by numbers in the grid (all costs are in dollars).

If a traveller wishes to leave the island, they must make their way to a port (any port will do). Travellers may only move horizontally and vertically between regions. For instance, a traveller beginning in the region marked “5” on the left hand side could leave via the upper port at a cost of $5 + 1 + 2 = 8$ dollars (leaving from a different port would be more expensive). In fact, 8 dollars is the most that any traveller needs to pay to leave the island, no matter where they begin.

Each of the following scenarios describes an island, its taxes and its ports. What is the greatest cost that a traveller must pay to leave the island from any region? You may assume that travellers will choose the cheapest possible route to a port. As an example, your answer for the scenario above would be 8.

8. 

9. 

10.
Senior Questions: Part A

Each question should be answered by a single choice from A to E. Questions are worth 3 points each.

1. Binary Coding

In a binary coding system, the letters A, B, C, D, E, F, G, and H are represented by $1, 10, 01, 11, 111, 101, 0111$ and $110$ respectively.

Which of the following patterns does not represent a string of the letters A, . . . , H in this system?

(A) $010110111$  (B) $01010011$  (C) $1110011110$

(D) $1100110011$  (E) $0111011001$

2. Tap

A tap has a complex set of pipes attached to it, as illustrated in the following picture. Each pipe has a maximum speed at which water can pass through it (written next to it in the diagram), measured in litres per second. Water may flow through the pipe at any speed up to this maximum, but it cannot flow faster.

You turn the tap on full, and water flows through the pipes as fast as possible. Note that at each junction the amount of water flowing in from above must equal the total amount of water flowing out below. How much water in total will flow out the bottom of the entire structure (measured again in litres per second)?

(A) 47  (B) 48  (C) 49  (D) 50  (E) 51
3. Guessing Game

Ben’s grandfather said to him “I have thought of a 3 digit number for you to guess. Each time you guess I will say ‘Too high’, ‘too low’ or ‘correct’. You have 9 guesses. By then you should know the number.”

Ben’s first 8 guesses were 600 (too high), 300 (too low), 450 (too high), 360 (too low), 405 (too low), 427 (too high), 416 (too high) and 410 (too high).

By this time Ben knew that the number must be between 406 and 409. But he only had one guess left and so could not be sure that he would know the number after his last guess.

Ben’s guessing strategy was flawed. After which guess was it no longer possible for him to be sure of knowing the correct number after his remaining guesses, assuming that he used the best strategy for them?

(A) 600  (B) 300  (C) 450  (D) 360  (E) 405
4. Robot

The weather is lovely, so — like any other sensible person — you decide to spend a day on the beach with your pet robot.

Your robot is fairly simple; all it can do is walk around the beach and trace patterns in the sand. It begins facing north and accepts the following instructions:

- \( F_x \): Walk forwards \( x \) centimetres.
- \( L_x \): Stay in the same place and turn \( x \) degrees to the left.
- \( R_n[...] \): Repeat whatever appears in the brackets \( n \) times.

For example, the robot can trace out an equilateral triangle in the sand using the instructions \( F30 \ L120 \ F30 \ L120 \ F30 \ L120 \) (see the illustration below). This same pattern can be simplified by writing \( R3 \ [ F30 \ L120 \] \), since the forward/left instructions are repeated three times.

\[ \begin{array}{c}
\quad \\
\end{array} \]

Note that if you repeat the forward/left instructions four times instead of three, the pattern in the sand would be exactly the same (the robot simply walks over one edge of the triangle twice).

You are feeling artistic, and you would like the robot to trace out the pattern illustrated below.

\[ \begin{array}{c}
\quad \\
\end{array} \]

You have given the robot the following instructions:

\[ R6 \ [ R12 \ [ F50 \ L60 \] \ L120 \] \]

Unfortunately it does not trace out the pattern that you wanted. With a slap of the forehead, you realise that the number in one of your instructions was wrong. Which instruction had the incorrect number?

\[ \begin{array}{c}
(A) \ R6 \quad (B) \ R12 \quad (C) \ F50 \quad (D) \ L60 \quad (E) \ L120 \\
\end{array} \]
5–7. Antarctic Exploration

You wish to prospect along a straight line joining two bases 50 km apart in Antarctica. You identify a number of potential sites for camps. Each camp site allows prospectors to explore 5 km in either direction. Prospectors can also explore 5 km from each base.

In the questions below, the first line is the location of potential camp sites, and the second is the cost of setting up the camp.

What is the smallest cost of setting up camps that will allow prospectors to explore the whole line?

<table>
<thead>
<tr>
<th>Location</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>28</th>
<th>32</th>
<th>36</th>
<th>40</th>
<th>44</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location</th>
<th>4</th>
<th>9</th>
<th>11</th>
<th>17</th>
<th>21</th>
<th>26</th>
<th>31</th>
<th>35</th>
<th>40</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location</th>
<th>5</th>
<th>9</th>
<th>14</th>
<th>20</th>
<th>25</th>
<th>29</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>39</th>
<th>45</th>
<th>46</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>
8–10. Game

You are playing a rather unusual game on a $4 \times 4$ grid, in which each square contains a number. You begin in the top left square of this grid, and you must travel to the bottom right square. The rules state that you must move either one square down or one square right in each turn.

To begin with you have a score of zero. Each time you move into a new square, you must halve your current score (rounding down if necessary) and then add the value of this new square. Your aim is to reach the bottom right square with the smallest score possible.

As an example, consider the following grid.

\[
\begin{array}{cccc}
3 & 9 & 6 \\
1 & 4 & 4 & 5 \\
8 & 2 & 5 & 4 \\
1 & 8 & 5 & 9 \\
\end{array}
\]

The smallest possible final score for this grid is 12, which is achieved as follows.

<table>
<thead>
<tr>
<th>Move</th>
<th>begin</th>
<th>down</th>
<th>right</th>
<th>down</th>
<th>right</th>
<th>right</th>
<th>down</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>1 4 2 5 4 7 7 12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Score</td>
<td>0 1 4 4 7 7 12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is the smallest possible score for the following grids?

8.  
\[
\begin{array}{ccc}
2 & 2 & 2 \\
4 & 6 & 6 \\
4 & 8 & 4 \\
4 & 8 & 2 \\
\end{array}
\]

9.  
\[
\begin{array}{ccc}
1 & 2 & 3 \\
4 & 3 & 2 \\
1 & 2 & 3 \\
4 & 3 & 2 \\
\end{array}
\]

10.  
\[
\begin{array}{ccc}
1 & 2 & 3 \\
4 & 5 & 6 \\
8 & 9 & 10 \\
12 & 13 & 14 \\
\end{array}
\]
**Answers**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Junior</td>
<td>B</td>
<td>E</td>
<td>A</td>
<td>E</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>18</td>
<td>27</td>
<td>51</td>
</tr>
<tr>
<td>Intermediate</td>
<td>C</td>
<td>C</td>
<td>E</td>
<td>A</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>Senior</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>E</td>
<td>16</td>
<td>15</td>
<td>23</td>
<td>5</td>
<td>3</td>
<td>23</td>
</tr>
</tbody>
</table>