Junior Questions: Part A

Each question should be answered by a single choice from A to E. Questions are worth 3 points each.

1. Garden Plots

A vegetable garden is laid out in hexagonal plots. In the diagram below, the shaded plots are growing tomatoes. Marigolds will be planted in some of the adjacent plots to deter pests. The number in each shaded plot is the number of adjacent plots that will be planted with marigolds.

Which number should be in plot A?

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

Solution

The plots containing marigolds can be determined as shown

There is 1 plot adjacent to plot A that is planted with marigolds. Hence (B).
2. SMS

In order to reduce the length of your SMS messages, you use the following rules:

- remove all spaces;
- remove all vowels (a, e, i, o, u);
- replace double letters with single letters.

For example, the message good morning would be sent as gdmrng, saving 6 characters (1 space, 4 vowels, 1 double letter).

You send the following three messages:

two white asters
five violet tulips
three red daisies

How many characters do you save in total?

(A) 23  (B) 25  (C) 27  (D) 29  (E) 31

Solution

Applying the rules to each phrase, we have:

two white asters → twowhiteasters → twwhtstrs → twhtstrs
a saving of $16 - 8 = 8$ characters,

five violet tulips → fiveviolettulips → fvvlttls → fvtlts
a saving of $18 - 7 = 11$ characters,

three red daisies → threereddaisies → thrrddss → thrds
a saving of $17 - 5 = 12$ characters.

The total saving is $8 + 11 + 12 = 31$ characters. Hence (E).
3. Cities 1

The land of Straightopia has four cities, all built along a single straight highway. The distances between the cities are as follows:

<table>
<thead>
<tr>
<th>Distance</th>
<th>City P</th>
<th>City Q</th>
<th>City R</th>
<th>City S</th>
</tr>
</thead>
<tbody>
<tr>
<td>City P</td>
<td>3 km</td>
<td>3 km</td>
<td>1 km</td>
<td></td>
</tr>
<tr>
<td>City Q</td>
<td>3 km</td>
<td>6 km</td>
<td>4 km</td>
<td></td>
</tr>
<tr>
<td>City R</td>
<td>3 km</td>
<td>6 km</td>
<td>2 km</td>
<td></td>
</tr>
<tr>
<td>City S</td>
<td>1 km</td>
<td>4 km</td>
<td>2 km</td>
<td></td>
</tr>
</tbody>
</table>

You are travelling along the highway from one end to the other. In which order might you travel past the four cities?

(A) Q, P, S, R  (B) Q, R, P, S  (C) R, P, S, Q
(D) S, R, P, Q  (E) S, R, Q, P

Solution

We start with the cities that are closest to each other, and then at each step choose the unselected city that is closest to one of the selected ones.

The cities that are closest to each other are P and S. So our order so far is P, S.

Then the city closer to P or S is R, and R is closer to S than to P. So now we have P, S, R.

Finally, Q is closer to P than to R, so our final order is Q, P, S, R.

So you would travel past the cities in the order Q, P, S, R (or R, S, P, Q). Hence (A).
4. Robot

The weather is lovely, so — like any other sensible person — you decide to spend a day on the beach with your pet robot.

Your robot is fairly simple; all it can do is walk around the beach and trace patterns in the sand. It begins facing north and accepts the following instructions:

- \( F_x \): Walk forwards \( x \) centimetres.
- \( L_x \): Stay in the same place and turn \( x \) degrees to the left.
- \( R_n \): Repeat whatever appears in the brackets \( n \) times.

For example, the robot can trace out an equilateral triangle in the sand using the instructions \( F30 \ L120 \ F30 \ L120 \ F30 \ L120 \) (see the illustration below). This same pattern can be simplified by writing \( R3 \ [ \ F30 \ L120 \ ] \), since the forward/left instructions are repeated three times.

Note that if you repeat the forward/left instructions four times instead of three, the pattern in the sand would be exactly the same (the robot simply walks over one edge of the triangle twice).

You are feeling artistic, and you would like the robot to trace out the pattern illustrated below.

You have given the robot the following instructions:

\[ R2 \ [ \ R4 \ [ \ F50 \ L90 \ ] \ L90 \ ] \]

Unfortunately it does not trace out the pattern that you wanted. With a slap of the forehead, you realise that the number in one of your instructions was wrong. Which instruction had the incorrect number?

(A) \( R2 \)  (B) \( R4 \)  (C) \( F50 \)  (D) The first \( L90 \)  (E) The second \( L90 \)

**Solution**

The instructions \( R4 \ [ \ F50 \ L90 \ ] \) cause the robot to trace out a square. The full set of instructions causes it to trace out two squares.

The starting directions (shown) are rotated by 90 degrees.

The desired pattern can be achieved by rotating the starting direction for the second square by 180 degrees rather than 90 degrees. The instructions should therefore be \( R2 \ [ \ R4 \ [ \ F50 \ L90 \ ] \ L180 \ ] \).

Hence (E).
Junior Questions: Part B

Each question should be answered by a number in the range 0–999.
Questions are worth 2 points each.

5–7. Borrowing Albums 1

A library has several different albums of a band. They are in a stack in order 1 2 3 \ldots N, with 1 on top. When an album is borrowed, the order of the remaining albums does not change. For instance, if \( N \) was 5 and album 3 was borrowed, the stack would become 1 2 4 5.

Several albums are borrowed, and a week later returned. As they are returned they are put on top of the stack in the order that they are returned.

For each of the following questions, the order shown is the order in the stack after the albums have been returned.

What is the smallest number of albums that has been borrowed?

5. \( 4 \ 3 \ 2 \ 1 \)

6. \( 3 \ 6 \ 2 \ 5 \ 1 \ 4 \)

7. \( 2 \ 4 \ 6 \ 8 \ 1 \ 3 \ 5 \ 7 \ 9 \)

Solution

Removed albums are returned to the top of the stack, so any unborrowed albums will be in order at the bottom of the stack.

*Question 5*

The stack ends in 1, so the other 3 albums must have been borrowed.

*Question 6*

The stack ends in 1 4, so the other 4 albums must have been borrowed.

*Question 7*

The stack ends in 1, 3, 5, 7, 9, so the other 4 albums must have been borrowed.
8–10. Rescue

It is a dark and stormy night, and you are watching over the troubled seas off the coast of Tasmania. You are leading a team of three rescue craft, stationed in the water and ready for any emergency.

The sea can be pictured as a grid of regions, such as the grid illustrated below. Your three rescue craft are stationed in three of these regions. Although they are swift and sturdy, they can only travel north, south, east and west through the grid (in particular, they cannot travel along diagonals). A rescue craft can move through one square in one minute.

Each region of the sea is protected by the rescue craft closest to it, as measured by the time it takes each craft to reach it. If a region is equally close to two rescue craft, it is protected by both of them. The diagram below shows the regions protected by each craft in the grid above.

Your task is to identify which rescue craft is protecting the most regions. In this case it is the second craft, which protects eight regions.

Each of the following scenarios describes the positioning of the three rescue craft within the sea. For each scenario, what is the largest number of regions protected by a single rescue craft? As an example, your answer for the scenario illustrated above would be 8.
Solution
The first step is to identify the regions that are protected by only one of the rescue craft. Then the remaining regions will be protected by two or more craft.
In the diagrams below, the numbered cells are protected by one rescue craft, the white cells are protected by two and the black regions are protected by all three.

Question 8
The three crafts protect
\[4 + 3 + 2 + 1 = 10,\]
\[3 + 4 + 2 + 1 = 10,\]
\[2 + 4 + 6 + 6 = 18\] regions.
Hence 18.

Question 9
The three crafts protect
\[6 + 5 + 4 + 3 + 2 + 1 = 21,\]
\[3 + 4 + 5 + 6 + 7 + 2 = 27,\]
\[1 + 2 + 3 + 4 + 5 + 8 = 23\] regions.
Hence 27.

Question 10
The three crafts protect
\[9 + 5 + 4 = 19,\]
\[4 + 5 + 9 + 8 + 7 + 6 + 6 + 6 = 51,\]
\[1 + 2 + 3 + 9 + 9 + 9 = 33\] regions.
Hence 51.
Intermediate Questions: Part A

Each question should be answered by a single choice from A to E. Questions are worth 3 points each.

1. Stars

Beginning with some number $n$, you write a line of ‘*’s by repeatedly applying the following rules:

- If $n$ is 0, stop.
- If $n$ is odd, write a single ‘*’ and reduce $n$ by 1.
- If $n$ is even, divide $n$ by 2.

For example, if you begin with $n = 3$ then you would proceed as follows. Since 3 is odd, you write a single ‘*’ and subtract one to give $n = 2$. Since 2 is even, you divide by two giving $n = 1$. Finally, since 1 is odd you write another ‘*’ and subtract one. Now $n = 0$ and you stop, having written two ‘*’s in total.

If you begin with the number $n = 77$, how many ‘*’s do you write in total?

(A) 2  (B) 3  (C) 4  (D) 5  (E) 6

Solution

The steps from 77 down to 0 are

$$\begin{align*}
77 & \rightarrow 76 \rightarrow 38 \rightarrow 19 \rightarrow 9 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 0
\end{align*}$$

In four of these and odd number is reduced by 1, so 4 stars are generated, hence (C).

2. Buried Treasure

The treasure has been buried, but the pirates do not trust each other. They agree that each of them will have part of the instructions needed to locate the treasure. They all know where to start, so the instructions are of the form “$m$ metres E (or W), $n$ metres N (or S)”.

After several hours of calculation, they produce the following set of instructions:

- Olaf: 100 metres E, 60 metres N
- Bluebeard: 100 metres E, 60 metres S
- Hook: 100 metres W, 60 metres N
- Sinbad: 100 metres W, 40 metres N
- Noah: 100 metres E, 40 metres N

Alas, despite all of their sums, they got it wrong. The location of the buried treasure is 200 metres E and 80 metres N. It can be found using 4 of the instructions, but not all 5. Whose instructions should not be in the set?

(A) Olaf  (B) Bluebeard  (C) Hook  (D) Sinbad  (E) Noah

Solution

A pirate who followed all instructions would end up 100 metres E, 140 metres N of the starting point. This is 100 metres W and 60 metres N of the treasure. So Hook’s instructions should not be in the set. Hence (C).
3. Cities 2

The land of Pitopia is centred upon a large circular lake. Around this lake is a circular highway, with five cities placed along the highway. The distances between the cities are as follows:

<table>
<thead>
<tr>
<th>Distance</th>
<th>City P</th>
<th>City Q</th>
<th>City R</th>
<th>City S</th>
<th>City T</th>
</tr>
</thead>
<tbody>
<tr>
<td>City P</td>
<td>5 km</td>
<td>3 km</td>
<td>6 km</td>
<td>4 km</td>
<td></td>
</tr>
<tr>
<td>City Q</td>
<td>5 km</td>
<td>2 km</td>
<td>1 km</td>
<td>3 km</td>
<td></td>
</tr>
<tr>
<td>City R</td>
<td>3 km</td>
<td>2 km</td>
<td>3 km</td>
<td>5 km</td>
<td></td>
</tr>
<tr>
<td>City S</td>
<td>6 km</td>
<td>1 km</td>
<td>3 km</td>
<td>2 km</td>
<td></td>
</tr>
<tr>
<td>City T</td>
<td>4 km</td>
<td>3 km</td>
<td>5 km</td>
<td>2 km</td>
<td></td>
</tr>
</tbody>
</table>

Note that there are always two different ways of travelling from one city to another (corresponding to the two different directions around the lake); the table above lists the shorter distance in each case.

You are travelling along the highway in a constant direction around the lake. In which order might you travel past the five cities?

(A) P, Q, S, T, R  (B) P, R, S, T, Q  (C) P, R, Q, T, S
(D) P, S, Q, T, R  (E) P, T, S, Q, R

Solution

We start with the cities that are closest to each other, and then at each step choose the unselected city that is closest to one of the selected ones.

The cities that are closest to each other are Q and S. So our order so far is Q, S.

Then the city closest to Q is R and the city closest to S is T. So now we have R, Q, S, T.

Finally, P is between R and T.

The only option consistent with this order around the lake is P, T, S, Q, R. Hence (E).
4. Robot

The weather is lovely, so — like any other sensible person — you decide to spend a day on the beach with your pet robot.

Your robot is fairly simple; all it can do is walk around the beach and trace patterns in the sand. It begins facing north and accepts the following instructions:

- \( F_x \): Walk forwards \( x \) centimetres.
- \( L_x \): Stay in the same place and turn \( x \) degrees to the left.
- \( R_n \[ \ldots \] \): Repeat whatever appears in the brackets \( n \) times.

For example, the robot can trace out an equilateral triangle in the sand using the instructions \( F_{30} \, L_{120} \, F_{30} \, L_{120} \, F_{30} \, L_{120} \) (see the illustration below). This same pattern can be simplified by writing \( R_3 \[ \ F_{30} \, L_{120} \ ] \), since the forward/left instructions are repeated three times.

Note that if you repeat the forward/left instructions four times instead of three, the pattern in the sand would be exactly the same (the robot simply walks over one edge of the triangle twice).

You are feeling artistic, and you would like the robot to trace out the pattern illustrated below.

You have given the robot the following instructions:

\[
R_3 \[ \ R_3 \[ \ F_{50} \, L_{120} \ ] \, L_{60} \ ]
\]

Unfortunately it does not trace out the pattern that you wanted. With a slap of the forehead, you realise that the number in one of your instructions was wrong. Which instruction had the incorrect number?

(A) The first \( R_3 \)    (B) The second \( R_3 \)    (C) \( F_{50} \)    (D) \( L_{120} \)    (E) \( L_{60} \)

**Solution**

The instructions \( R_3 \[ \ F_{50} \, L_{120} \ ] \) cause the robot to trace out a triangle (as above). The full set of instructions causes it to trace out only three triangles.

The starting directions (shown) are rotated by 60 degrees.

The desired pattern can be achieved by drawing three more triangles, keeping the same rotation of the starting direction.

The instructions should therefore be \( R_6 \[ \ R_3 \[ \ F_{50} \, L_{120} \ ] \, L_{60} \ ] \).

Hence (A).
Intermediate Questions: Part B

Each question should be answered by a number in the range 0–999. Questions are worth 2 points each.

5–7. Borrowing Albums 2  

A library has several different albums of a band. They are in a stack in order 1 2 3 … N, with 1 on top. When an album is borrowed, the order of the remaining albums does not change. For instance if N was 5 and album 3 was borrowed, the stack would become 1 2 4 5.

Several albums are borrowed, and a week later returned. When they are returned they are put anywhere in the stack. In the example above, if album 3 was returned the stack could be 3 1 2 4, 1 3 2 4, 1 2 3 4, or 1 2 4 3.

For each of the following questions, the order shown is the order in the stack after the albums have been returned.

What is the smallest number of albums that could have been borrowed?

5. 4 2 5 6 1 3 7
6. 4 2 3 6 5 7 8 1
7. 2 4 3 1 5 8 6 7 9

Solution

Whenever an album is borrowed, the remaining albums are still in increasing order. Finding the smallest set of albums borrowed corresponds to finding the largest set of albums in increasing order. This can be done by inspection. A more systematic way is to apply Dynamic Programming. At each stage we add an album, and the position it would have if it was part of the largest set. This will be 1 more than the largest position of albums with smaller initial order. The largest of these positions gives the maximum number of unborrowed albums, from which the number borrowed can be easily calculated.

**Question 5**  
album 4 2 5 6 1 3 7  
position 1 1 2 3 1 2 4  
There could be at most four unborrowed items (4 or 2, 5, 6 and 7), with three (2 or 4, 1 and 3) being borrowed. Hence 3.

**Question 6**  
album 4 2 3 6 5 7 8 1  
position 1 1 2 3 3 4 5 1  
There could be at most five unborrowed items (2, 3, 6 or 5, 7, 8), with three (4, 5 or 6, and 1) being borrowed. Hence 3.

**Question 7**  
album 2 4 3 1 5 8 6 7 9  
position 1 1 2 1 3 4 4 5 6  
There could be at most six unborrowed items (2, 3, 5, 6, 7 and 9), with three (4, 1 and 8) being borrowed. Hence 3.
8–10. Ports

An island is divided into several regions, as illustrated on the grid below. A few of these regions are ports; these are marked with shaded squares. Each of the remaining regions charges a small tax for travellers who pass through the region; these taxes are indicated by numbers in the grid (all costs are in dollars).

If a traveller wishes to leave the island, they must make their way to a port (any port will do). Travellers may only move horizontally and vertically between regions. For instance, a traveller beginning in the region marked “5” on the left hand side could leave via the upper port at a cost of $5 + 1 + 2 = 8$ dollars (leaving from a different port would be more expensive). In fact, 8 dollars is the most that any traveller needs to pay to leave the island, no matter where they begin.

Each of the following scenarios describes an island, its taxes and its ports. What is the greatest cost that a traveller must pay to leave the island from any region? You may assume that travellers will choose the cheapest possible route to a port. As an example, your answer for the scenario above would be 8.

8.  

9.  

10. 
Solution

We need to know the cost of travelling to the nearest port from each cell. Rather than calculating this cell by cell, it is easier to work out from each port. But we need to be a little careful.

Consider the scenario

\[
\begin{array}{ccc}
3 & 2 & 1 \\
4 & 2 & 1 \\
\end{array}
\]

The squares adjacent to the port have no additional squares to go through, so their cost is just the cost for the square.

If we fill in the squares next to the ones most recently filled, we get

\[
\begin{array}{ccc}
2 & 1 \\
5 & 3 & 1 \\
\end{array}
\]

and then

\[
\begin{array}{ccc}
5 & 2 & 1 \\
7 & 3 & 1 \\
\end{array}
\]

This gives correct results. In particular, the cheapest possible route from the top left square to the port is 5.

However, if the scenario was

\[
\begin{array}{ccc}
5 & 4 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

we would get the incorrect result

\[
\begin{array}{ccc}
9 & 4 & 1 \\
3 & 2 & 1 \\
\end{array}
\]

So instead we use Djikstra’s algorithm, giving

\[
\begin{array}{ccc}
5 & 4 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

\[
\begin{array}{ccc}
4 & 1 \\
2 & 1 \\
3 & 2 & 1 \\
\end{array}
\]

\[
\begin{array}{ccc}
8 & 4 & 1 \\
3 & 2 & 1 \\
\end{array}
\]

and for the sample scenario

\[
\begin{array}{ccc}
1 & 2 & 1 \\
5 & 2 & 3 & 1 \\
6 & 4 & 2 & 2 \\
2 & 4 \\
\end{array}
\]

\[
\begin{array}{ccc}
2 & 1 & 1 \\
3 & 2 \\
3 & 2 \\
2 & 4 \\
\end{array}
\]

\[
\begin{array}{ccc}
3 & 2 & 1 \\
4 & 3 & 2 \\
6 & 6 & 2 \\
2 & 4 \\
\end{array}
\]

\[
\begin{array}{ccc}
3 & 2 & 1 \\
4 & 3 & 2 \\
6 & 6 & 2 \\
2 & 4 \\
\end{array}
\]

Applying this algorithm to the questions gives

**Question 8**

\[
\begin{array}{ccc}
3 & 2 & 1 \\
4 & 5 & 3 \\
2 & 3 & 4 \\
2 & 4 \\
\end{array}
\]

Greatest cost = 5

**Question 9**

\[
\begin{array}{ccc}
1 & 3 & 2 \\
1 & 3 & 5 \\
3 & 1 & 4 \\
1 & 1 & 3 \\
\end{array}
\]

Greatest cost = 5

**Question 10**

\[
\begin{array}{ccc}
12 & 9 & 5 \\
13 & 19 & 4 \\
8 & 11 & 6 \\
4 & 7 & 9 \\
\end{array}
\]

Greatest cost = 16
Senior Questions: Part A

Each question should be answered by a single choice from A to E. Questions are worth 3 points each.

1. Binary Coding  
2010 S.2

In a binary coding system, the letters A, B, C, D, E, F, G, and H are represented by 1, 10, 01, 11, 111, 101, 0111 and 110 respectively.
Which of the following patterns does not represent a string of the letters A, . . . , H in this system?

(A) 010110111  (B) 01010011  (C) 111001110

(D) 1100110011  (E) 0111011001

Solution

The pattern in (A) is easily decoded to CCBE. (There are other decodings.) However the pattern in (B) contains an 00 pair, and as there is no letter whose code contains 00, the pattern must split into 01010 and 011. 001 can be decoded to CA, but as there is no letter whose code ends in 010, the pattern 01010 must be split into 010 and 10. Again, 010 must be split into 0 and 10. No letter is coded as 0, so the pattern 11001010 cannot be decoded into a string of letters. Hence (B).
2. Tap

A tap has a complex set of pipes attached to it, as illustrated in the following picture. Each pipe has a maximum speed at which water can pass through it (written next to it in the diagram), measured in litres per second. Water may flow through the pipe at any speed up to this maximum, but it cannot flow faster.

You turn the tap on full, and water flows through the pipes as fast as possible. Note that at each junction the amount of water flowing in from above must equal the total amount of water flowing out below. How much water in total will flow out the bottom of the entire structure (measured again in litres per second)?

(A) 47 (B) 48 (C) 49 (D) 50 (E) 51

Solution

Consider the 3rd level pipe with maximum speed 14 litres per second. It feeds into two pipes with maximum speeds 3 and 8 litres per second. As the amount of water flowing in must equal the amount of water flowing out, the maximum network speed of this pipe is 11 litres per second. However the pipe beside it of maximum speed 12 litres per second is not limited by the speeds of the pipes leading out of it. So its maximum network speed remains at 12 litres per second.

Working from the bottom up, the maximum network speeds of the pipes in the network are as shown below

The top level pipe has a maximum network speed of 47 litres per second. Hence (A).
3. Guessing Game

2009 S.6

Ben’s grandfather said to him “I have thought of a 3 digit number for you to guess. Each time you guess I will say ‘Too high’, ‘too low’ or ‘correct’. You have 9 guesses. By then you should know the number.”

Ben’s first 8 guesses were 600 (too high), 300 (too low), 450 (too high), 360 (too low), 405 (too low), 427 (too high), 416 (too high) and 410 (too high).

By this time Ben knew that the number must be between 406 and 409. But he only had one guess left and so could not be sure that he would know the number after his last guess.

Ben’s guessing strategy was flawed. After which guess was it no longer possible for him to be sure of knowing the correct number after his remaining guesses, assuming that he used the best strategy for them?

(A) 600  (B) 300  (C) 450  (D) 360  (E) 405

Solution

The correct guessing strategy is binary search. That is, to guess the number in the middle of the range. If this strategy is followed, one guess is enough for a range of 3 numbers, two guesses for a range of 7 numbers and 3 (4, 5, 6, 7, 8, 9) guesses is enough for a range of 15 (31, 63, 127, 255, 511, 1023) numbers. But 1 (2, 3, 4, 5, 6, 7, 8, 9) guesses is not enough for a range of 4 (8, 16, 32, 64, 128, 256, 512, 1024) or more numbers.

For Ben’s guesses we have

<table>
<thead>
<tr>
<th>Guesses left</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>999</td>
<td>599</td>
<td>599</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>100</td>
<td>100</td>
<td>301</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>900</td>
<td>500</td>
<td>299</td>
<td></td>
</tr>
<tr>
<td>Guess</td>
<td>600</td>
<td>300</td>
<td>…</td>
<td></td>
</tr>
</tbody>
</table>

After Ben’s guess of 300, he had narrowed the range to 299 numbers. But by then he only had 7 guesses left and could no longer be sure of guessing the number. Hence (B).
4. Robot

The weather is lovely, so — like any other sensible person — you decide to spend a day on the beach with your pet robot.

Your robot is fairly simple; all it can do is walk around the beach and trace patterns in the sand. It begins facing north and accepts the following instructions:

- $F_x$: Walk forwards $x$ centimetres.
- $L_x$: Stay in the same place and turn $x$ degrees to the left.
- $R^n[\ldots]$: Repeat whatever appears in the brackets $n$ times.

For example, the robot can trace out an equilateral triangle in the sand using the instructions $F_{30} \ L_{120} \ F_{30} \ L_{120} \ F_{30} \ L_{120}$ (see the illustration below). This same pattern can be simplified by writing $R_3[\ F_{30} \ L_{120} \ ]$, since the forward/left instructions are repeated three times.

Note that if you repeat the forward/left instructions four times instead of three, the pattern in the sand would be exactly the same (the robot simply walks over one edge of the triangle twice).

You are feeling artistic, and you would like the robot to trace out the pattern illustrated below.

You have given the robot the following instructions:

$R_6[\ R_{12} [\ F_{50} \ L_{60} \ ] \ L_{120} ]$

Unfortunately it does not trace out the pattern that you wanted. With a slap of the forehead, you realise that the number in one of your instructions was wrong. Which instruction had the incorrect number?

(A) $R_6$  (B) $R_{12}$  (C) $F_{50}$  (D) $L_{60}$  (E) $L_{120}$
Solution

The instructions R12 [ F50 L60 ] cause the robot to trace out a hexagon (twice). The full set of instructions causes it to trace out three hexagons in the order 1, 2, 3. (As each hexagon is traced twice the order is really 1, 1, 2, 2, 3, 3.)

The starting directions (shown) are rotated by 120 degrees.
The desired pattern requires another three hexagons, superimposed on the first three. The starting point will be the same, but with different starting directions.

This can be achieved by rotating the starting direction by 60 degrees instead of 120. The instructions should therefore be R6 [ R12 [ F50 L60 ] L60 ]. Hence (E).

The hexagons will then be traced in the order 1, 4, 2, 5, 3, 6. The steps are shown below, with a dot at the starting point.
5–7. **Antarctic Exploration**

You wish to prospect along a straight line joining two bases 50 km apart in Antarctica. You identify a number of potential sites for camps. Each camp site allows prospectors to explore 5 km in either direction. Prospectors can also explore 5 km from each base.

In the questions below, the first line is the location of potential camp sites, and the second is the cost of setting up the camp. What is the smallest cost of setting up camps that will allow prospectors to explore the whole line?

<table>
<thead>
<tr>
<th>Location</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>28</th>
<th>32</th>
<th>36</th>
<th>40</th>
<th>44</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location</th>
<th>4</th>
<th>9</th>
<th>11</th>
<th>17</th>
<th>21</th>
<th>26</th>
<th>31</th>
<th>35</th>
<th>40</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location</th>
<th>5</th>
<th>9</th>
<th>14</th>
<th>20</th>
<th>25</th>
<th>29</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>39</th>
<th>45</th>
<th>46</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
Solution

At each stage we add a camp. As we add each camp, we find the lowest total setting up cost for that camp plus the camps that have to be set up between it and the first base. We will call this the running cost, and it is the state in this solution. The cost of the cheapest set of camps will be the smallest running cost of the camps within 10 km of the base.

To see how the state is used, consider location 12 in the first question. Location 12 is within 10 km of locations 4 and 8, so either could be used with it. The running cost of location 8 is smaller than that of location 4, so it is chosen and the running cost of location 12 is $2 + 1 = 3$.

**Question 5**

The camps that cover the whole distance with smallest cost are at 8, 12, 20, 28, 32, and 40, with a total cost of 14.

**Question 6**

The camps that cover the whole distance with smallest cost are at 9, 17, 26, 31, and 40, with a total cost of 15.

**Question 7**

The camps that cover the whole distance with smallest cost are at 9, 14, 20, 25, 33 or 34, 39, and 45, with a total cost of 23.
8–10. Game

You are playing a rather unusual game on a $4 \times 4$ grid, in which each square contains a number. You begin in the top left square of this grid, and you must travel to the bottom right square. The rules state that you must move either one square down or one square right in each turn.

To begin with you have a score of zero. Each time you move into a new square, you must halve your current score (rounding down if necessary) and then add the value of this new square. Your aim is to reach the bottom right square with the smallest score possible.

As an example, consider the following grid.

The smallest possible final score for this grid is 12, which is achieved as follows.

<table>
<thead>
<tr>
<th>Move</th>
<th>begin</th>
<th>down</th>
<th>right</th>
<th>down</th>
<th>right</th>
<th>right</th>
<th>down</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Score</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>7</td>
<td>7</td>
<td>12</td>
</tr>
</tbody>
</table>

What is the smallest possible score for the following grids?

8. 

9. 

10.

Solution

We label each cell with the smallest score to reach that cell. For the example in the question,

In the solutions below, the smallest score path is shaded. (There may be more than one.)

Question 8

Hence 5.

Question 9

Hence 3.

Question 10

Hence 22.