a The small cubes have at most three faces exposed. The only small cubes that have exactly three faces exposed are the 8 on the corners of the $10 \times 10 \times 10$ cube. The only small cubes that have exactly two faces exposed are the 8 on each edge of the $10 \times 10 \times 10$ cube that are not on its corners. All other cubes have less than two faces exposed. There are 12 edges on the $10 \times 10 \times 10$ cube, so the number of small cubes removed at the first step is $8 + (12 \times 8) = 104$.

b Alternative i

At the first step the 8 small cubes at the corners are removed. The other 192 cubes come equally from the 12 edges but not from the corners. So the number of non-corner small cubes on each edge is $192/12 = 16$. Hence each edge in the original cube had a total of 18 small cubes.

Alternative ii

A cube with $n$ small cubes along one edge will lose $8 + 12(n - 2)$ small cubes at the first step. So $8 + 12(n - 2) = 200$, $12(n - 2) = 192$, $n - 2 = 16$, $n = 18$. Thus the original cube was $18 \times 18 \times 18$.

Alternative iii

From Part a, a $10 \times 10 \times 10$ cube loses 104 small cubes at the first step. Increasing the cube’s dimension by 1 increases the number of lost cubes by 12, one for each edge. Since $200 - 104 = 96$ and $96/12 = 8$, the cube that loses 200 small cubes at the first step is $18 \times 18 \times 18$.

c Alternative i

At the first step, each of the 6 faces of the $9 \times 9 \times 9$ cube are converted to a $7 \times 7$ single layer of small cubes. The exposed surface area of this layer is $7 \times 7$ small faces plus a ring of $4 \times 7$ small faces. So the surface area of the remaining object is $6 \times (49 + 28) = 6 \times 77 = 462$.

Alternative ii

First remove the middle small cube on one edge of the $9 \times 9 \times 9$ cube. This increases the surface area by 2 small faces. Then remove the small cubes either side. This does not change the surface area. Continue until only the two corner cubes remain. At this stage the surface area has increased by 2 small faces. Repeating this process on all 12 edges increases the surface area by $12 \times 2$ small faces. At this stage all 8 corner cubes have 6 exposed faces. So removing the 8 corner cubes reduces the surface area by $8 \times 6$ small faces. The surface area of the original $9 \times 9 \times 9$ cube was $6 \times 81 = 486$. Hence the surface area of the remaining object is $486 + 24 - 48 = 462$.

d At the first step, the original $9 \times 9 \times 9$ cube is reduced to a $7 \times 7 \times 7$ cube with a $7 \times 7$ single layer of small cubes placed centrally on each face. At this stage the number of small cubes removed is $8 + (12 \times 7) = 92$.  

continued
At the second step the only small cubes removed are the edge cubes in the $7 \times 7$ single layers. This leaves a $7 \times 7 \times 7$ cube with a $5 \times 5$ single layer of small cubes placed centrally on each face. So in this step, the number of small cubes removed is $6(4 + 4 \times 5) = 144$.

At the third step the only small cubes removed are the edge cubes in the $5 \times 5$ single layers and the edge cubes in the $7 \times 7 \times 7$ cube. So in this step, the number of small cubes removed is $6(4 + 4 \times 3) + 8 + (12 \times 5) = 164$. Thus the number of small cubes remaining is $(9 \times 9 \times 9) - 92 - 144 - 164 = 329$. 