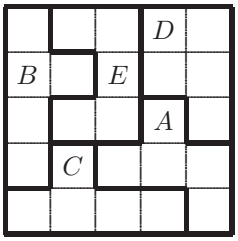
DIRICHLET STUDENT SAMPLE PROBLEMS

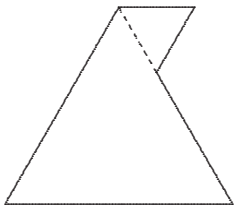
PROBLEM 1

A 5×5 grid is made from five pentominoes and some squares have letters written in them as shown. Fill the remaining squares so that every pentomino and every row and column of the grid contains each of the letters A, B, C, D, E exactly once. Give your reasoning.



PROBLEM 2

A pentagon is formed from an equilateral triangle by attaching a smaller equilateral triangle to one of its sides as shown.



(a) Show in two different ways how to cut a single piece from this pentagon to produce two different polygons that will tessellate separately.

(b) For each polygon from Part (a), draw two different tessellations that show at least 12 copies of the polygon.

PROBLEM 3

(a) Determine the base in which each of the following calculations was done.

$$243 \times 2 = 1041 \qquad 323 \times 3 = 1302$$

(b) Determine the base in which each of the following calculations was done.

$$432 \times 22 = 11724 \qquad 341 \times 23 = 11503$$

PROBLEM 4

Sue's favourite number is 6, the more the better. She made a big number consisting of lots of consecutive 6s followed by a 7 (her second favourite number) and squared it. What is the maximum number of 6s Sue can have in her number if the sum of the digits in its square is less than 2008?

PROBLEM 5

Each working day Bahir normally rides his bike 3 km from home to the railway station, takes the 36 minute train trip to town, and then rides his bike 2 km downhill from the station to his work place at an average speed of 24 km/h. The train averages 100 km/h and the first bike ride takes 9 minutes.

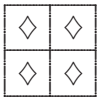
(a) What distance is the train trip?

(b) What is the average speed for Bahir's trip from home to work?

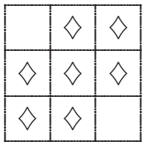
(c) One day Bahir drove his car to work. The trip was 2 km shorter but the average speed was 36 km/h slower. How much longer did this car trip take?

PROBLEM 6

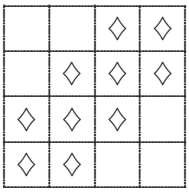
Deborah drew a sequence of square grid patterns with diamonds in some of the small squares as shown.



S_1



S_2



S_3

(a) Draw S_4.

(b) How many diamonds are there in S_5?

(c) How many diamonds are there in S_n?

(d) How many small squares in S_n do not contain diamonds?

PROBLEM 7

Trevor is playing *Money Trail* in a TV show. There is a 5×5 square grid of glass panels on the studio floor with dollar amounts written on them and lights underneath. He starts on the panel marked 'S' and when the bell rings he steps from one panel to the next and tries to reach the panel marked 'F' before the buzzer goes. At each step Trevor is only allowed to step to the next panel up or to the right. If he does this correctly the light under the panel he moves to flashes on and off. If he breaks this rule a hooter blows, all the lights go out and he has to leave the game. If he reaches 'F' in time he collects all the money he stepped on and the path of panels that would have given the maximum amount lights up.

\$10	\$30	\$60	\$10	F
\$70	\$20	\$10	\$20	\$70
\$10	\$20	\$80	\$10	\$10
\$30	\$10	\$50	\$20	\$30
S	\$20	\$10	\$90	\$10

(a) What is the maximum amount of money Trevor can win?

(b) Show on the grid the path(s) Trevor must take to win the maximum amount of money.

PROBLEM 8

There is a rule for the decimal expansion of $\frac{1}{m}$, regardless of whether m is prime or not:

the number of digits in the repetend divides the number of integers from 1 to m whose highest common factor with m is 1.

For example $\frac{1}{21} = 0.\overline{047619}$, the number of digits in the repetend is 6, and there are 12 integers from 1 to 21 whose highest common factor with 21 is 1: 1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20.

(a) Check this rule for $\frac{1}{51}$.

(b) Find the family of fractions amongst $\frac{n}{51}$ for $n = 1, 2, 3, ..., 50$, that have the same digits in their repetends as $\frac{1}{51}$ but cyclically rotated.

(c) Check the rule for all other families of fractions amongst $\frac{n}{51}$ for $n = 1, 2, 3, ..., 50$, that have the same digits in their repetends but cyclically rotated.