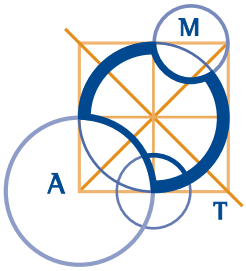

NOETHER STUDENT SAMPLE PROBLEMS

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PROBLEM 1

Automatic Teller Machines in Marchhareland dispense notes of size 30, 40 and 50 rallods. To buy himself a new hat, Mad Hatter has to send a 30 rallods banknote to a factory. What is the largest amount he can withdraw that is certain to contain at least one 30 rallods note?

PROBLEM 2

A positive integer N greater than 100 is written on a blackboard. When its two rightmost digits are erased, the number which is left is $N \div 134$. Find all possible numbers N.

PROBLEM 3

Simplify:

$$\frac{12a^3 + 4a^2 - 3ab^2 - b^2}{6a^2 - 3ab - 2a + b} \div \frac{9a^2 + 6a + 1}{9a^2 - 1}\,.$$

PROBLEM 4

Simplify:

$$\frac{2x^2 - 3xy + 4x - 6y}{8x^3 - 27y^3} \times \frac{4x^2 + 6xy + 9y^2}{x^2 + x - 2}\,.$$

PROBLEM 5

Sum the series:

$$1^2 - 3^2 + 5^2 - 7^2 + 9^2 - 11^2 + \cdots + 10001^2 - 10003^2\,.$$

PROBLEM 6

When a positive integer X is written in base 9, it is a three-digit number. When X is written in base 6, it is a three-digit number that consists of the same digits as the first one but in reverse order. Find the decimal representations of all such numbers X.

PROBLEM 7

Find all real numbers x such that

$$x^4 \leq 8x^2 - 16.$$

PROBLEM 8

On each of the faces of a cube a real number is written. For each of the faces, the number written on it equals the product of the numbers written on all the faces that have a common side with this face. What is the sum of all the numbers that are written on the faces of the cube? Find all possible answers.

PROBLEM 9

A bus and a truck left a town at the same time and went along the same road at constant speeds of 60 km/h and 70 km/h respectively. Some time later, a car set off along the same road at a constant speed of 90 km/h. How much time passed between the moments when the truck and the car set off, if the car overtook the bus exactly half an hour before the car caught up with the truck?

PROBLEM 10

Prove that for any positive integer n, the value of the expression

$$5 \times 2^{3n-2} + 3^{3n-1}$$

is divisible by 19.

PROBLEM 11

The side length of an equilateral triangle ABC is 5. P and Q are points on the sides AB and BC respectively such that $BP = 2$ and $BQ = 3$. Find $\angle PAQ + \angle PCQ$.

PROBLEM 12

Points A, B, C and D lie on the circumference of a circle. The chords AC and BD meet at the point X. Prove that if $\angle ABC = \angle BCD$, then $AX = XD$.

PROBLEM 13

In a quadrilateral $ABCD$, $\angle ABD = 70°$, $\angle CAD = 42°$ and $\angle CDA = 68°$. Find the size of the angle $\angle CBD$.

PROBLEM 14

Points A, B and C lie on the circumference of a circle. The straight line which is tangent to the circle at A meets the line BC extended at E. A point D is placed on BC so that AD bisects the angle BAC. Prove that $AE = DE$.

PROBLEM 15

Helen runs faster than Jane but slower than Lisa. They started at the same time from the same place and ran around a circular route. After a while they stopped simultaneously at the same place from which they started. It turned out that Lisa overtook Jane ten times during this run. How many times did it occur that one of the girls overtook another? Assume that each of the girls ran at a constant speed. (The final stop together is not counted as an overtake.)

PROBLEM 16

Positive integers x and y satisfy the equation

$$3x + 5y = 2xy - 1.$$

What is the value of $x - y$? Find all possible answers.