Gauss Enrichment Stage

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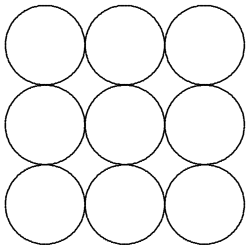
Chapter 1.
And These are a Few of My Favourite Problems – *Norm Hoffman*

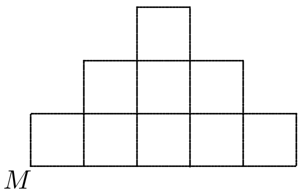
1. A farmer and his wife, with their son and daughter and dog, were going to town. They came to a river. Now the only boat was a frail one which could not hold more than 70 kg. The farmer and his wife each weighed 70 kg, the son and daughter each weighed 35 kg and the dog weighed 10 kg. How did they all get across the river?

2. The nine balls shown here touch each other. Three of them are red, three are white and three are blue. They are arranged so that each red ball touches a white ball, each white ball touches a blue ball and each blue ball touches a red ball. Show how the balls may be coloured.

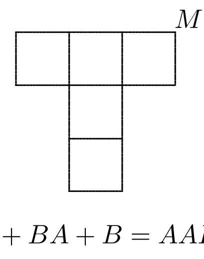


3. In each case, divide the figure into two parts of equal area by a single line drawn through the point M.

(a)



(b)



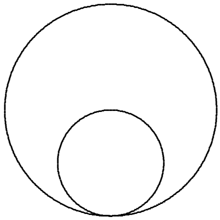
4. Solve the cryptogram $AB + BA + B = AAB$.

5. (a) At what time between 1pm and 2pm are the hands of a clock pointing in exactly the same direction?

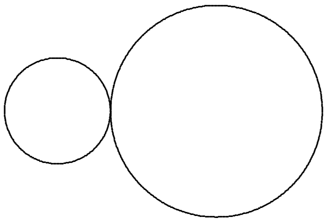
   (b) At what time between 1pm and 2pm are the hands of a clock pointing in exactly opposite directions?

6. The diameter of the large circle is twice that of the small. How many rotations will the small circle make in rolling, without slipping

   (a) round the inside of the large circle;



   (b) round the outside of the large circle?



7. With how few bearers can an explorer make a six-day march across an absolutely barren desert if he and the available bearers each carry only enough food and water to last one man four days?

8. It is commonly held that most people remember the faces of their friends more readily than their names. Dr. Watson decided to test this hypothesis at the next meeting he chaired. He found that he recalled the names and faces of 50% of those present. He recalled the faces of 80% and the names of 63 of those present. Given that he recalled the name, the face or both of everyone present at the meeting, how many people (excluding Dr. Watson) do you reckon were at the meeting?

9. There was once a market gardener who drove to the market to sell his 30 watermelons at a rate of three for a dollar. On his way he passed the market garden of a friend who asked him to take

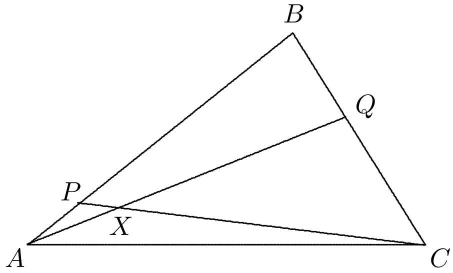
his 30 watermelons along and sell them too, but at a rate of two for a dollar. The first gardener agreed. However, to make the job easier he decided to make a single batch of both his and his friend's melons to sell every five melons for two dollars. And that is exactly what he did.

On his way home, he paid his friend the 15 dollars due to him. But when he was about to deliver to his wife his share in the sale, he realised to his amazement that only nine dollars were left to him instead of ten. Since he was sure he hadn't spent a whole dollar on drinks he cudgelled his brain to find out what had become of the missing dollar. He never did find out, but you might have been able to help the poor fellow.

10. It takes a cyclist three minutes to ride a kilometre on level ground against the wind, but only two minutes to return with the wind behind him. How long would it take him to ride one kilometre on a calm day?

Now try Problem 1 in the Gauss Student Problems Book.

Chapter 2.
Parallels



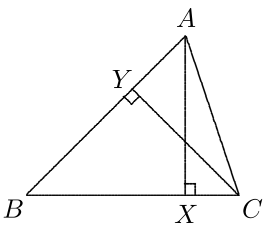
P and Q lie on the sides AB and BC respectively of the triangle ABC such that $\frac{AP}{PB} = \frac{1}{4}$ and $\frac{BQ}{QC} = \frac{2}{3}$. The area of triangle ABC is 85 cm^2. PC and QA intersect at X. Find the area of triangle AXC and the ratios $\frac{AX}{XQ}$ and $\frac{CX}{XP}$.

Comparing the Areas of Triangles

In this section we see that we can use areas to investigate certain situations. For example:

Any side of the triangle may be treated as the base. But we must use the corresponding perpendicular height or altitude.

If $AB = 20$, $BC = 24$ and $AX = 10$ in this diagram, we can determine CY by working out the area in two ways.



$\frac{1}{2}BC \times AX = \frac{1}{2}AB \times CY$.
$\frac{1}{2} \times 24 \times 10 = \frac{1}{2} \times 20 \times CY$.
Thus $CY = 12$.

The significant factor in comparing the areas of triangles is that we rarely have to compute the actual areas or even find the length of an altitude.

There are two particular situations in which this is the case:

(i) Problems in which parallel lines are included in the figure.

(ii) Problems in which triangles have a common vertex.