

Newton Enrichment Stage

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Chapter 1

The Castle of Problems

They passed it every day on their way to school and every afternoon on their way home. Its enormous walls, with many windows at odd levels to each other, kept them curious. The light that always appeared in the castle tower, day and night, kept them wondering. The howling of the wind around the castle filled the street with mystery.

It was mid afternoon. Jasper and Anastasi had been dismissed early from their school sports carnival.

‘Anastasi, I see a door open around the side of the old castle. This is our chance to look inside. Come on, follow me,’ whispered Jasper as she crept through the garden and moved slowly into the castle. They were barely inside the door when it slammed shut behind them. Within a flash they turned to flee but from the inside ... there was no sign of a door!

‘Jasper, what have we done? We may never find our way out of here!’ cried Anastasi.

‘This is a problem ! No one knows where we are. How will they ever think to look for us here?’ said Jasper in despair.

At that moment Anastasi noticed letters waving around in a 3D hieroglyphic image on the wall.

.SMELBORP FO ELTSAC EHT FO TUO
YAW RUOY DNIF LLIW UOY DNA
OG UOY SA SMELBORP EHT EVLOS
.TI OT REWSNA EHT WONK UOY ECNO
MELBORP A REGNOL ON SI MELBORP A

‘Smelborp?’ questioned Jasper. ‘This makes no sense! Could it be another language?’

‘Is it like the hieroglyphics used by the Egyptians – you know, some kind of code?’ asked Anastasi.

‘The letters are like those of the English language.’

‘It could be a code of English or a Latin language, or some alien language from another planet!’ exclaimed Anastasi with excitement.

‘It’s a great imagination you have, Anastasi, but then again, sometimes you need the weirdest ideas to help you find the answer! Let’s look for

more clues', she said as they turned back to the wall and read it piece by piece.

'Yes, weird ideas and a wild imagination *can* help to solve a problem.' said Anastasi before he directed his attention back to the message. 'It has a dot right at the start, and yet no full stop at the end.'

'Perhaps the start is really the end? Try it backwards – does it make any sense?'

As they began to read the message from end to start it became clear to them!

'A PROBLEM IS NO LONGER A PROBLEM WHEN YOU KNOW THE ANSWER TO IT. SOLVE THE PROBLEMS AS YOU GO AND YOU WILL FIND YOUR WAY OUT OF THE CASTLE OF PROBLEMS' they read.

'Hey, that's right you know,' said Jasper. 'If you know how to solve a problem, then – no problem! Okay, there is hope. We can do this Anastasi, we can find our way out of here.'

'Let's make a pact. No matter what happens, we stay calm, think positively and don't give up!' reassured Anastasi.

They placed one hand upon the other and promised they would stick to this plan.

Suddenly the wall behind them slid apart. They walked through the opening only to find themselves in a room surrounded by large computer touch screens.

IN THE AGNESI ROOM

'Wow! These are huge!' cried Anastasi with excitement and as he swept his hand over them, he saw the images of his hand movement on the screen.

'Hey, look at this!' cried Jasper as one of the screens illuminated and the first problem appeared.

Many staircases lead into rooms.
Many staircases lead out of rooms.
Many rooms can be entered.
Some rooms can be exited.

Choose wisely, which path you will take around the castle,
as there may be a door which lets you into a room,
but once in it you will not be able to go out the door you came in!

The Gauss Room has a staircase leading to the Fibonacci Room.
The Noether Room has a staircase leading to the Fibonacci Room.
The Agnesi Room has a staircase leading to the Noether Room.
The Polya Room has a staircase leading to the Gauss Room.

You can go to the Noether Room by stairs from the Gauss Room.
You can go to the Euler Room by stairs from the Fibonacci Room.
You can go to the Polya Room by stairs from the Agnesi Room.
You can go to the Noether Room by stairs from the Polya Room.

You are now in the Agnesi Room.

They read and read and read. Room after room after room: staircase this way, staircase that way. They knew they needed some kind of strategy to get through this problem! Questions came firing into their heads.

‘How many different rooms are there?’

‘How are we going to keep track of all these staircases?’

‘Jasper, there are six different room names and so there are six rooms, but the staircases are one-way. I sure hope the doors are labelled!’

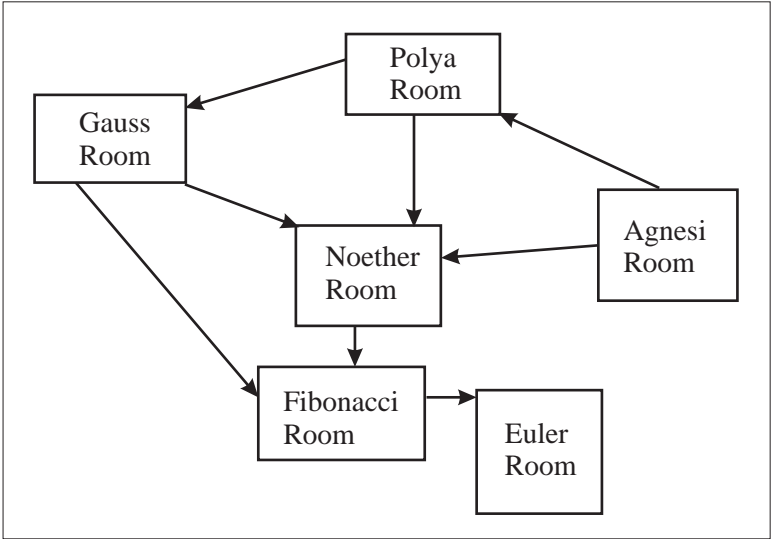
‘Perhaps you can use your drawing skills,’ said Jasper with excitement, as she ran to the wall.

Anastasi did not hesitate. Within seconds, he had drawn six boxes in different places on the wall each containing a room name.

Jasper read aloud ‘The Gauss Room has a staircase which leads to the Fibonacci Room.’

‘Okay’ said Anastasi as he drew a line from the Gauss box to the Fibonacci box. ‘This arrow points to the Fibonacci box. It means we can get into the Fibonacci Room if we follow the staircase from the Gauss Room!’

As Jasper continued to read the paths, Anastasi furiously filled in his diagram. Within minutes they stood back and marvelled at his artistic map of the castle rooms!



‘We don’t know which room has the exit so we’ll need to find a path which goes into each room.’

They examined the map.

CAN YOU HELP THEM TO FIND A PATH THAT WILL TAKE THEM THROUGH EVERY ROOM IN THE CASTLE?

‘Drawing a diagram has been a great way to start solving this problem,’ said Anastasi.

‘If we take the order Agnesi, Polyva, Noether, Fibonacci, Euler, we miss the Gauss room! If we take the order Agnesi, Polyva, Gauss, Fibonacci, Euler, we miss the Polyva Room. If we take the order Agnesi, Polyva, Gauss, Noether, Fibonacci, Euler, we’ve covered every room. That’s our path!’ exclaimed Jasper.

Anastasi found the print icon on the large screen and within seconds a printout came floating from the ceiling with their maps recorded on it. Suddenly two doors appeared in the screens of the walls. One door was labelled *To the Noether Room*, the other *To the Polyva Room*. Following the path they had chosen they approached the door labelled Polyva.

‘Jasper, there is no handle on this door,’ said Anastasi, as he place his hands on the door to push it. To his surprise, his hands passed through

the door. He bravely stepped through. Jasper followed and they found themselves climbing a narrow staircase.

Their hearts raced as they took each step, wondering what they would encounter in the next room. The castle was filled with a musty smell. What a strange place, with its old stone walls and yet surprisingly modern technology!

IN THE POLYA ROOM

‘I hope this is right’ said Jasper as she pushed open the door at the top of the staircase and they entered. The door closed slowly behind them. They looked around the Polya Room.

They were standing on a square stone, with ‘Edinburgh’ written on it. It was one of many stones in a line going in both directions to the left and right away from the one they stood on. None of the others had names on them. There was a message on the wall:

TO OPEN THE DOORS LEADING TO THE GAUSS AND NOETHER ROOMS FOLLOW THESE INSTRUCTIONS:

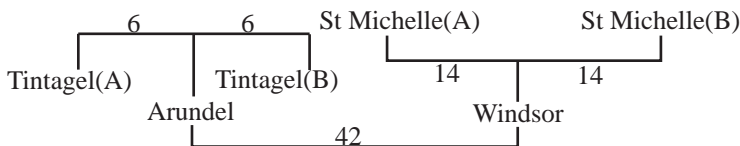
‘Edinburgh’, ‘Windsor’, ‘Arundel’, ‘Tintagel’ and ‘St Michelle’ are all on the line of stones, but not necessarily in that order.

Arundel is 42 stones from Windsor, while Windsor is 14 stones from St Michelle. Tintagel is 6 stones away from Arundel. St Michelle is 50 stones from Tintagel. Edinburgh is 40 stones from St Michelle and 54 stones from Windsor.

Go to the Arundel stone and stamp your foot three times.

‘The stones are all named after castles. How are we going to solve this problem and decide which stone we must stand on?’ complained Jasper.

‘We made a pact, remember,’ said Anastasi. ‘A diagram might be useful.’ He quickly started to draw.



‘Here’s what we get from the first three clues,’ he said. ‘We have to put in two positions for some castles because we don’t know enough yet.’

‘Well, if St Michelle is 50 stones from Tintagel, then St Michelle must be in position B and Tintagel must be in position A so that the numbers add to 50.’