Ramanujan Enrichment Stage

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Chapter 1: Penguin Estimation

Sliding down the icy slopes was so much fun for the feathered friends. Sliding on her tummy, Mali, always the bravest penguin of the four, was first to reach the bottom of the icy hill.

Levi and Telia both slid down, always eager to beat the other. Squirt tumbled head first and came crashing into the other three who had stopped in front of a massive building.

Squirt got up, flattened his messed up feathers and joined the others staring in amazement.

Finally they had reached the Mawson station. Led by their curiosity and mischievous nature, the friends had travelled many kilometres to find the station to learn more about science.

'Quick, everyone, follow me.' Mali led the way. With their backs against the wall and taking sideward steps, together they moved around the building until they came to a door.

With quick beak movements left and right to check no one was watching, the friends snuck into the station, avoiding being seen by the scientists. Finally they arrived at an empty room.

They found themselves in front of a large screen. A film showed large groups of penguins marching over the ice against ferocious winds.

'This is amazing', said Mali. 'I'm sure that is my mum there.'

'I don't know—she looks the same as my mum', exclaimed Squirt.

'Where are they marching to?' asked Levi.

'This is the trek our mums took from the colony site, after we were hatched. They marched 112 kilometres to the ocean shore to collect food. We were slowly hatching in our eggs while our dads watched over us', explained Mali.

'Wow! How long did that 112 kilometres trek take? That's a huge distance. I think it would take me a year to get there', said Levi.

'Five days. I think I could do it in five days!' remarked a confident Squirt.

'That's impossible Squirt. Go on, show us how you can do it.'

'Hmm, well . . . perhaps . . . ' Squirt replied, now slightly embarrassed.

'To save you taking the trek Squirt, why don't we estimate how long it would take?' interrupted Telia.

'How would you estimate it without me actually doing the trek?' asked Squirt, keen to save himself the journey, as he now watched the penguins battling the harsh wind conditions on the screen in front of him.

'If we measure how far you can waddle in a short time, we can calculate your waddle speed and then use this to estimate a time for completing the trek . . . with an error factor, of course', explained Telia.

'Great, quick! We need a clock and a distance measuring device', said Levi, keen to prove Squirt incorrect.

The penguins scattered in search of the tools they would need, jumping on desks, peeking into cupboards and scrabbling in drawers.

'Found a measuring tape', called out Mali.

'I've got a stop watch', added Levi.

'Right, ready to go. Squirt, stand on the edge of the wall here, and when I say "go", start to waddle', instructed Telia. 'Ready, set, go!' She clicked the stop watch.

Squirt, keen to meet the target he had set, waddled as quickly as possible in the one minute until Telia called out 'stop'. Levi and Mali quickly measured the distance and found he had covered a surprising 12.3 metres.

'That's a good effort Squirt, but you may get quite tired over a long trek. So now you need to repeat this 20 times so we can see what happens as you get tired.' Squirt waddled for another 19 sets of one minute and all the distances were recorded.

| 12.3 | 12.3 | 12.1 | 11.8 | 11.5 | 11.4 | 11.3 | 11.2 | 11.1 | 11.1 |
| 11.1 | 11.0 | 11.0 | 10.7 | 10.5 | 10.4 | 10.3 | 10.3 | 10.3 | 10.3 |

Levi, who was good at mental arithmetic, quickly found the sum of the 20 values to be 222.

'Easy, now we divide 222 by 20 giving 11.1 metres as the average distance I waddle in 1 minute', explained Squirt.

'Well, maybe, . . . but I can see that over the 20 trials your distance kept decreasing as you got tired', said Levi. 'Our average would be slightly more accurate if you repeated that 100 times.'

'Let's work with the 11.1 metres per minute for now', said Mali. 'Where

do we go from here?'

'Well, 11.1 metres per minute is the same as 111 metres per 10 minutes', said Telia.

'Or 0.111 kilometres per 10 minutes, as there are 1000 metres in a kilometre', calculated Mali.

'True', said Squirt. 'Now, if we multiply both of these by 1000 we have 111 kilometres in 10 000 minutes. This is close to the 112 kilometres which we know is the length of the journey. Now we need to change 10 000 minutes to days.'

'We need to divide it by 60 to change the minutes to hours, and then divide it by 24 to change the hours to days', said Levi, keen to get to the answer.

'Dividing by 60 is the same as multiplying by $\frac{1}{60}$', said Mali.

They calculated $\frac{10000}{1} \times \frac{1}{60} \times \frac{1}{24} = \frac{1000}{1} \times \frac{1}{6} \times \frac{1}{24} = \frac{125}{1} \times \frac{1}{6} \times \frac{1}{3} = \frac{125}{18}$, which is approximately 7.

'We could have found an approximate answer more quickly by using 25 instead of 24', said Mali. 'Then $10000 \div 25 = 400$ and $\frac{400}{60} = \frac{20}{3}$ or $6\frac{2}{3}$, which is approximately 7 days.'

'Squirt, you were actually very close, well done', said Levi.

'Although we didn't factor in the extreme weather conditions. You would have been waddling against winds of 70 kilometres per hour. And knowing you, you would have had many rest breaks!' exclaimed Mali.

'We also didn't factor in that I won last year's Antarctic Toboggan Championship, so would have covered a fair bit of the distance on my belly!' remarked Squirt.

'Guys, we also could have done the calculations by changing Squirt's waddle speed from 11.1 metres per minute, to $11.1 \times 60 \times 24 = 15\,984$ metres per day, or 15.984 kilometres per day, which is approximately 16 kilometres per day. Then $112 \div 16 = 7$, so the answer is approximately 7 days!' Telia was so pleased with these calculations she had not noticed the other three had already left to investigate another room. Quickly she waddled off to find them!

Exercises

As our penguin friends **estimated** answers from the best data available, these problems are about **estimation**, not about finding exact answers. Because you may round differently or at a different stage in the problem, your answers may not be exactly the same as those given in the solutions on page 64.

1. To find 48×36, Mary first rounds the values to the nearest 10 and then multiplies them, that is, she calculates 50×40. Which of the following is true?

Mary's estimate is

(a) less than the exact answer

(b) equal to the exact answer

(c) more than the exact answer.

2. A round-the-world yacht race is approximately 40000 kilometres long.

(a) A blue whale travels at an average speed of $23\,\text{km/h}$. By rounding the whale's speed to the nearest 10 km/h, estimate the number of hours it would take to travel the same distance as the yacht race.

(b) Is this slightly more or less than the exact answer?

(c) Find the approximate number of days it would take the whale to travel the distance by converting the number of hours from your estimate in (a) to days, by dividing by 25 (instead of 24).

(d) Is this slightly more or less than the answer would have been if you had divided by 24?

3. Each year Alice loved to attend the annual Christmas Pageant Parade in Adelaide, watching the many floats, clowns and performers travel along the main roads of the city centre. Last year she was curious to estimate the number of people who had also come along to watch. Rows of adults and children had lined both sides of the route. Many children sat in the front rows. Then adults sat in deck chairs in rows, some with children in their arms. More adults stood at the back, some with children on their shoulders.

Alice stood on the route, facing the crowd. She estimated a 1-metre section of the route and counted a surprising total of 34 adults and