



AUSTRALIAN MATHS TRUST

Maths Challenge Junior: Years 7–8 Practice Problem

J2: The Price is Perfect

Solutions

- 1 The number is a multiple of each of its digits (from the second property). Then the digit 0 cannot be in the number since any multiple of 0 is 0.

Since the number has seven digits, at least two are even since no two digits are the same (from the first property). Multiples of even numbers are even, so the number itself is even. Then the digit 5 cannot be in the number since any multiple of 5 ends in a 0 or 5 – the number cannot end in 0 (0 is already excluded) and it cannot end in 5 (it has to be even).

The possible digits left are 1, 2, 3, 4, 6, 7, 8 and 9. Suppose 9 is not in the number. The sum of the remaining seven digits is $1+2+3+4+6+7+8 = 31$. Now 3 is in the number, but any multiple of 3 has a digit sum which is a multiple of 3 and 31 is not a multiple of 3. So these seven digits cannot form the number and hence 9 must be in the number.

Now the sum of all eight digits is $1+2+3+4+6+7+8+9 = 40$. Since any multiple of 9 has a digit sum which is a multiple of 9, whichever digit is excluded must leave a digit sum divisible by 9 (since 9 is in the number). The multiple of 9 closest to (and less than) 40 is 36, that is, 4 less. In fact, the only way to get a digit sum divisible by 9 is to exclude the digit 4 – leaving out any other digit cannot give a digit sum divisible by 9. So the digit 4 cannot be in the number.

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2 The number contains the digits 1, 2, 3, 6, 7, 8 and 9 in some order. It is guaranteed to divide by 9 exactly since its digit sum is 36; it follows that it is also divisible by 3. We already know it has to be even, so it will also divide by 2 and hence 6. It obviously divides by 1 exactly, so that just leaves the digits 7 and 8. We only need to look for the largest number which divides exactly by $7 \times 8 = 56$.

We start at the largest number using the digits permitted (remembering that it must be even) and systematically work our way down through a list of possibilities (only even numbers being considered).

Possible Number	div by 7	div by 8	div by 56
9876312	No	Yes	No
9876132	Yes	No	No
9873612	Yes	No	No
9873216	No	Yes	No
9873162	No	No	No
9873126	No	No	No
9872316	No	No	No
9872136	No	Yes	No
9871632	No	Yes	No
9871362	No	No	No
9871326	No	No	No
9871236	No	No	No
9867312	Yes	Yes	Yes

So the largest 7-digit number with the two properties is 9867312.

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