AUSTRALIAN MATHS TRUST

Maths Challenge
Intermediate: Years 9–10
Practice Problem

I1: Dice Duels

Solutions

a **i** The probability of David rolling a 3 on his die is $1/6$.
The following table shows the possible scores Corey may get from his two counters.

+	1	2
1	2	3
2	3	4

Since the four outcomes from the two counters are equally likely, the probability Corey scores 3 is $2/4 = 1/2$.
So the probability David and Corey both score 3 in the same game is 1/6 x 1/2 = 1/12.
Since the players can also draw, the probability of David losing is less than 1/2. **[1]**

Alternative i

Corey will lose if his score is less than David's.

The following table shows each score S that Corey may flip, its probability from Part **i**, and the probability that David rolls a number greater than S.

Corey's score S	Probability of S	David's possible scores $> S$	Probability David scores $> S$
2	1/4	3, 4, 5, 6	4/6
3	2/4	4, 5, 6	3/6
4	1/4	5, 6	2/6

Thus Corey's total probability of losing is
$(\frac{1}{4})(\frac{1}{\ })(1 \times 4 + 2 \times 3 + 1 \times 2) = \frac{1}{24}(12) = \frac{1}{\ }$.

less than $\frac{1}{2}$. Hence, Corey is more likely to lose the game. **[1]**

Alternative ii

The following table shows each score N that David may roll, Corey's possible scores less than N, and their probability from Part **i**.

David's number N	Corey's scores $< N$	Probability Corey scores $< N$
1	none	0
2	none	0
3	2	1/4
4	2, 3	3/4
5	2, 3, 4	4/4
6	2, 3, 4	4/4

Since the probability of each value of N is $1/6$, Corey's probability of losing is

$(\frac{1}{6})(\frac{1}{4})(0+0+1+3+4+4) = \frac{1}{24}(12) = \frac{1}{2}$.

Since the players can also draw, the probability of David losing is less than $1/2$. Hence, Corey is more likely to lose the game. **1**

Alternative iii

The following table shows whether Corey will lose (L), draw (D), or win (W) for each pair of Corey's and David's outcomes.

Scores	1	2	3	4	5	6
$1+1=2$	W	D	L	L	L	L
$1+2=3$	W	W	D	L	L	L
$2+1=3$	W	W	D	L	L	L
$2+2=4$	W	W	W	D	L	L

The probability of each entry is the same. There are more Ls than Ws, so Corey is more likely to lose the game. **1**

b Alternative i

The following table shows each score S that Corey may flip and its probability.

Corey's score S	Flips (total 8)	Probability of S
3	111	1/8
4	112, 121, 211	3/8
5	122, 212, 221	3/8
6	222	1/8

Corey will win if his score is greater than David's.

The next table shows each score S that Corey may flip, its probability, and the probability that David rolls less than S.

Corey's score S	Probability of S	David's possible scores $< S$	Probability David scores $< S$
3	1/8	1, 2	2/6
4	3/8	1, 2, 3	3/6
5	3/8	1, 2, 3, 4	4/6
6	1/8	1, 2, 3, 4, 5	5/6

Thus Corey's probability of winning is
$(\frac{1}{8})(\frac{1}{6})(1 \times 2 + 3 \times 3 + 3 \times 4 + 1 \times 5) = \frac{1}{48}(28) = \frac{7}{12}$.
Since $\frac{7}{12} > \frac{1}{2}$, Corey is more likely to win the game. **1**

Alternative ii

The following table shows each score N that David may roll, Corey's possible scores greater than N, and their probability from Alternative **i**.

David's number N	Corey's scores $> N$	Probability Corey scores $> N$
1	3, 4, 5, 6	8/8
2	3, 4, 5, 6	8/8
3	4, 5, 6	7/8
4	5, 6	4/8
5	6	1/8
6	none	0

Since the probability of each value of N is 1/6, Corey's probability of winning is
$(\frac{1}{6})(\frac{1}{8})(8 + 8 + 7 + 4 + 1 + 0) = \frac{1}{48}(28) = \frac{7}{12}$.
Since $\frac{7}{12} > \frac{1}{2}$, Corey is more likely to win the game. **1**

Alternative iii

The following table shows whether Corey will lose (L), draw (D), or win (W) for each pair of Corey's and David's outcomes.

Scores	1	2	3	4	5	6
$1+1+1=3$	W	W	D	L	L	L
$1+1+2=4$	W	W	W	D	L	L
$1+2+1=4$	W	W	W	D	L	L
$2+1+1=4$	W	W	W	D	L	L
$1+2+2=5$	W	W	W	W	D	L
$2+1+2=5$	W	W	W	W	D	L
$2+2+1=5$	W	W	W	W	D	L
$2+2+2=6$	W	W	W	W	W	D

The probability of each entry is the same. There are more Ws than Ls, so Corey is more likely to win the game. **1**

c Similar calculations to those in Part **a** show that an 8-faced die is more likely to win against one or two counters. So we try three counters.

Alternative i

Corey will win if his score is greater than David's.

The following table shows each score S that Corey may flip, its probability from Part **b** Alternative **i**, and the probability that David rolls less than S.

Corey's score S	Probability of S	David's possible scores $< S$	Probability David scores $< S$
3	1/8	1, 2	2/8
4	3/8	1, 2, 3	3/8
5	3/8	1, 2, 3, 4	4/8
6	1/8	1, 2, 3, 4, 5	5/8

Thus Corey's probability of winning is

$(\frac{1}{8})(\frac{1}{8})(1\times 2+3\times 3+3\times 4+1\times 5)=\frac{1}{64}(28)=\frac{7}{16}$.

The probability of a draw is

$(\frac{1}{8})(\frac{1}{8})(1\times 1+3\times 1+3\times 1+1\times 1)=\frac{1}{64}(8)=\frac{2}{16}$.

Hence the probability of David winning is $1-\frac{7}{16}-\frac{2}{16}=\frac{7}{16}$.

So the game is fair if Corey flips three counters. **1**

Alternative ii

The following table shows each score N that David may roll, Corey's possible scores greater than N, and their probability from Alternative **i**.

David's number N	Corey's scores $> N$	Probability Corey scores $> N$
1	3, 4, 5, 6	8/8
2	3, 4, 5, 6	8/8
3	4, 5, 6	7/8
4	5, 6	4/8
5	6	1/8
6	none	0
7	none	0
8	none	0

Since the probability of each value of N is 1/8, Corey's probability of winning is

$(\frac{1}{8})(\frac{1}{8})(8+8+7+4+1+0) = \frac{1}{64}(28) = \frac{7}{16}$.

The probability of a draw is

$(\frac{1}{8})(\frac{1}{8})(0+0+1+3+3+1+0+0) = \frac{1}{64}(8) = \frac{2}{16}$.

Hence the probability of David winning is $1 - \frac{7}{16} - \frac{2}{16} = \frac{7}{16}$.

So the game is fair if Corey flips three counters. $\boxed{1}$

Alternative iii

The following table shows whether Corey will lose (L), draw (D), or win (W) for each pair of Corey's and David's outcomes.

Scores	1	2	3	4	5	6	7	8
$1+1+1=3$	W	W	D	L	L	L	L	L
$1+1+2=4$	W	W	W	D	L	L	L	L
$1+2+1=4$	W	W	W	D	L	L	L	L
$2+1+1=4$	W	W	W	D	L	L	L	L
$1+2+2=5$	W	W	W	W	D	L	L	L
$2+1+2=5$	W	W	W	W	D	L	L	L
$2+2+1=5$	W	W	W	W	D	L	L	L
$2+2+2=6$	W	W	W	W	W	D	L	L

The probability of each entry is the same. The number of Ws and Ls are the same, so the game is fair. $\boxed{1}$