AUSTRALIAN MATHS TRUST

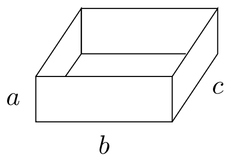
Maths Challenge
Junior: Years 7–8
Practice Problem

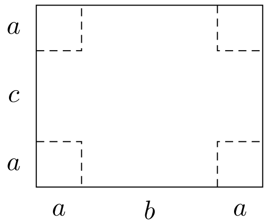
J1: Open Trays

Solutions

a The piece of cardboard is square, so the tray has a square base. The tray volume is the area of its base times its height. The only square numbers that divide 18 are 1 and 9. If the base of the tray is 1×1, then its height is 18, which is not allowed. If the base of the tray is 3×3, then its height is 2 cm. So the cutouts are 2 cm $\times$ 2 cm and the cardboard is 7 cm $\times$ 7 cm. **1**

b The required volume is $729 = 3^6$. So the only possible tray dimensions are the collections of three factors whose product is 729. The tray height, hence cut length, must be the smallest factor in each set. If the tray is $a \times b \times c$ with smallest factor a, then the original cardboard size was $(2a + b) \times (2a + c)$.





The following table gives all tray dimensions and the corresponding cardboard sizes.

tray	cardboard
$1 \times 1 \times 729$	3×731
$1 \times 3 \times 243$	5×245
$1 \times 9 \times 81$	11×83
$1 \times 27 \times 27$	29×29
$3 \times 3 \times 81$	9×87
$3 \times 9 \times 27$	15×33
$9 \times 9 \times 9$	27×27

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c Alternative i

First we factorise 3360 into prime factors: $3360 = 2^5 \times 3 \times 5 \times 7$. Hence the factors of 3360 up to 20 are: 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 15, 16, 20. The only set of three consecutive integers in this list whose product is 3360 is {14, 15, 16}. The product of any set of three consecutive integers greater than 16 is larger than 3360. So the cutouts were 14 cm × 14 cm and the original cardboard was 43 cm × 44 cm. **1**

Alternative ii

Experimenting with a calculator shows that $14 \times 15 \times 16 = 3360$. Any other three consecutive integers are, in order, either less than or greater than 14, 15, 16 respectively. Hence their product is less than or greater than 3360. So the cutouts were 14 cm × 14 cm and the original cardboard was 43 cm × 44 cm. **1**

Discussion

1. This problem is a modification of one proposed by Lorraine Mottershead.
2. It involves factorisation and finding volumes of cuboids (rectangular prisms).

Extensions

1. Two trays, each the shape of a cube, were made from square pieces of cardboard. Their volumes differed by 218 cm^3. What was the size of the two pieces of cardboard and the cutouts?
2. What is the greatest tray volume Alex can get from a 21 cm × 30 cm piece of cardboard?
3. What is the least and most amount of wasted cardboard if a tray of volume 3360 cm^3 is made from a sheet of cardboard with dimensions 34 cm × 38 cm?

Solutions to Extensions

1. For the tray to be the shape of a cube, the edge length of the cutouts and the edge length of the tray base must be the same. So the edge length of the original piece of cardboard is a multiple of 3. The following table lists the tray volume for original cardboard edge lengths from 3 to 30 cm.