AUSTRALIAN MATHS TRUST

Maths Challenge
Intermediate: Years 9–10
Practice Problem

I2: Squared Fractions

Solutions

1. If $\frac{1}{72}$ can be written with only two terms, then one of the terms must be $\frac{1}{81}$, $\frac{1}{100}$ or $\frac{1}{121}$, since

$$\frac{1}{72} - \frac{1}{144} = \frac{1}{144}$$

and any term used which is smaller than $\frac{1}{144}$ must be paired with one which is bigger than $\frac{1}{144}$.

We note

$$\frac{1}{72} - \frac{1}{81} = \frac{1}{8.9^2},$$

which is not the reciprocal of a square.

Similarly

$$\frac{1}{72} - \frac{1}{100} = \frac{7}{1800}$$

and

$$\frac{1}{72} - \frac{1}{121} = \frac{49}{(72)(121)},$$

neither of which is the reciprocal of a square.

Thus $\frac{1}{72}$ cannot be written in the desired form.

2. $\frac{1}{72} = \frac{1}{12^2} + \frac{1}{15^2} + \frac{1}{20^2}$.

3. $\frac{1}{8} = \frac{1}{3^2} + \frac{1}{12^2} + \frac{1}{15^2} + \frac{1}{20^2}$.

[This is the best answer, i.e. using fewest terms. Other answers are possible using five or more terms. That it is not possible to write $\frac{1}{8}$ using three terms requires an argument similar to the one in **1.**, but this is not required for full marks.]