AUSTRALIAN MATHS TRUST

Maths Challenge
Junior: Years 7–8
Practice Problem

J3: Fredholl Numbers

Solutions

a A Fredholl number must have at least two digits. The first three Fredholl numbers are 10, 12, 13. Hence 13 is the smallest Fredholl number that is prime.

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b If the distinct digits of a 6-digit Fredholl number are a and b, then the sum of all 6 of its digits is $a + a + a + b + b + b$. Since this sum is divisible by 3 the Fredholl number is also divisible by 3. Hence all, 6-digit Fredholl numbers are composite.

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c A 4-digit Fredholl number is one of three types.

Type 1. The first two digits are the same and the last two digits are the same. For example 3377. Then 11 divides the first and second halves of the Fredholl number exactly. So the Fredholl number is divisible by 11.

Type 2. The first and third digits are the same and the second and last digits are the same. For example 3737. Then the first and second halves of the Fredholl number are the same. So the Fredholl number is divisible by its first half.

Type 3. The first and last digits are the same and the second and third digits are the same. For example 3773. Then the sum of the first and third digits is the same as the sum of the second and last digits. Hence, by the divisibility rule for 11, the Fredholl number is divisible by 11. Hence all 4-digit Fredholl numbers are composite.

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d Alternative i

Qsnnmqc rfc bgqrglar bgegrq md _l 6+bgegr Dpcbfmjj lsk`cp _pc *a* _lb *b*,

Gd *a* _lb *b* _pc `mrf ctcl* rfcl rfc Dpcbfmjj lsk`cp gq ctcl* fclac amknmqgrc, Rfcpc _pc /. qsaf n_gpq &.* 0'* &.* 2'* &.* 4'* &.* 6'*&0* 2'* &0* 4'* &0* 6'* &2* 4'* &2* 6'* &4* 6',

Gd _ gq ctcl _lb ` gq 3* rfcl rfc Dpcbfmjj lsk`cp gq cgrfcp ctcl mp bgtgqg`jc `w 3* fclac amknmqgrc, Rfcpc _pc 3 qsaf n_gpq &.* 3'*&0* 3'* &2* 3'* &4* 3'* &6* 3',

Gd *a*) *b* gq _ ksjrgnjc md 1* rfcl rfc qsk md _jj 6 bgegrq gl rfc Dpcbfmjj lsk`cp gq _ ksjrgnjc md 1, Qm rfc Dpcbfmjj lsk`cp gq _ ksjrgnjc md 1* fclac amknmqgrc, Cvajsbgle n_gpq _jpc_bw jgqrcb* rfcpc _pc // qsaf n_gpq &.* 1'* &.* 7'* &/* 0'* &/* 3'* &/* 6'* &0* 5'* &1* 4'* &1* 7'* &3* 5'*&4* 7'* &5* 6',

Gd *a* ; . _lb *b* ; 5* rfcl rfc Dpcbfmjj lsk`cp gq bgtgqg`jc `w 5* fclac amknmqgrc,

Rfsq uc f_tc 05 n_gpq md bgegrq rf_r a_llmr k_ic _l 6+bgegr npgkc Dpcbfmjj lsk`cp,

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Alternative ii

Rfc dmjjmugle r_`jc jgqrq n_gpq md bgegrq dmp ughaf _jj Dpcbfmjj lsk`cpq _pc amknmqgrc, Gd 1 bgtgbcq rfc qsk md rfc rum bgegrq* rfcl 1 bgtgbcq rfc qsk md _jj 6 bgegrq _lb rfcpcdmpc 1 bgtgbcq rfc Dpcbfmjj lsk`cp,

Smaller digit	Larger digit	Reason all composite	Number of pairs
0	2 to 9	divisible by 2 to 9 respectively	8
1	2, 5, 8	3 divides sum of digits	3
2	4, 6, 8	divisible by 2	3
	5	divisible by 2 or 5	1
	7	3 divides sum of digits	1
3	6, 9	3 divides sum of digits	2
4	5	divisible by 2 or 5	1
	6, 8	divisible by 2	2
5	6, 8	divisible by 2 or 5	2
	7	3 divides sum of digits	1
6	8	divisible by 2	1
	9	3 divides sum of digits	1
7	8	3 divides sum of digits	1

Thus we have 27 pairs of digits that cannot make an 8-digit prime Fredholl number.

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