AMC PRACTICE QUESTIONS AND SOLUTIONS

Junior

1. 2014 J5

The value of $\dfrac{1}{0.04}$ is

(A) 15 (B) 20 (C) 25 (D) 40 (E) 60

▶ $\dfrac{1}{0.04} = \dfrac{1 \times 100}{0.04 \times 100} = \dfrac{100}{4} = 25,$

hence (C).

2. 2014 J10

Consecutive numbers are written on five separate cards, one on each card. If the sum of the smallest three numbers is 60, what is the sum of the largest three numbers?

(A) 62 (B) 63 (C) 64 (D) 65 (E) 66

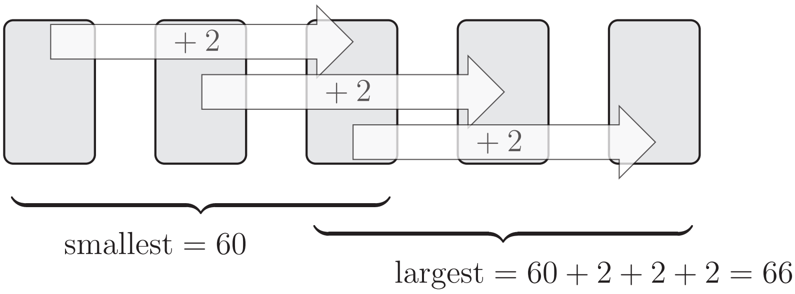
▶ *Alternative 1*

The average of the three smallest is 20, so they will be 19, 20 and 21. Then the largest three numbers are 21, 22 and 23, which add to 66,

hence (E).

Alternative 2

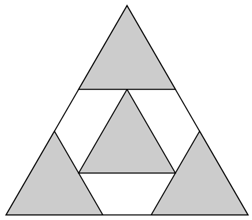
Comparing the largest three to the smallest three:



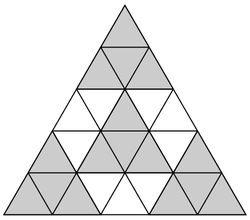
hence (E).

3. 2014 J15

Four equilateral triangles of the same size are arranged with horizontal bases inside a larger equilateral triangle, as shown. What fraction of the area of the larger triangle is covered by the smaller triangles?



(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{4}{9}$ (D) $\frac{4}{7}$ (E) $\frac{16}{25}$

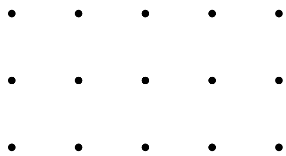
▶ 

The shortest length in the figure is half the side of one of the shaded triangles, so we use this length as the basis of the triangular grid shown.
Then 16 out of the 25 equal triangles are shaded,

hence (E).

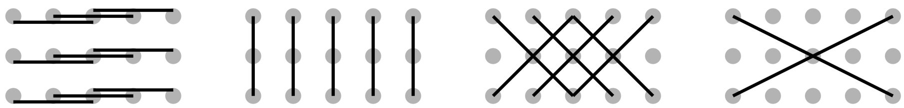
4. 2014 J20

A 3 by 5 grid of dots is set out as shown. How many straight line segments can be drawn that join two of these dots and pass through exactly one other dot?



(A) 14 (B) 20 (C) 22 (D) 24 (E) 30

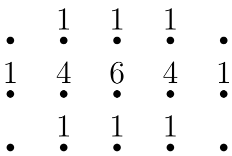
▶ *Alternative 1*
We draw all such line segments—horizontal, vertical, at 45°, and others:



$$9 + 5 + 6 + 2 = 22,$$

hence (C).

Alternative 2
The line segments can be classified by the midpoint dot, since then each line segment is counted only once. Also the number of line segments through each dot form a symmetric pattern of numbers:



There are 22 line segments,

hence (C).

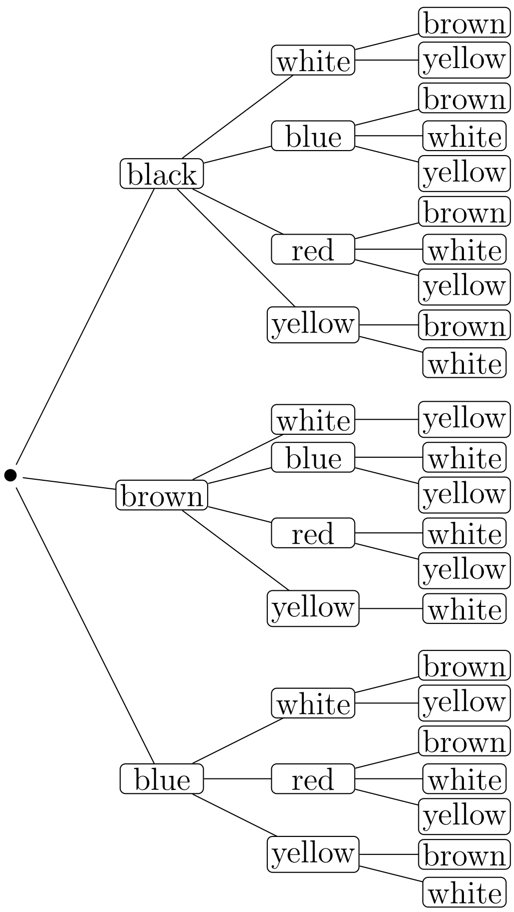
5. 2014 J25

Zac has three jackets, one black, one brown and one blue. He has four shirts, one white, one blue, one red and one yellow. He has three pairs of trousers, one brown, one white and one yellow. How many combinations of jacket, shirt and trousers are possible if no two items are of the same colour?

(A) 23 (B) 25 (C) 26 (D) 27 (E) 29

▶ *Alternative 1*

This tree shows all the possibilities of choosing jacket, then shirt, then trousers, each of a different colour:



hence (A).

Alternative 2

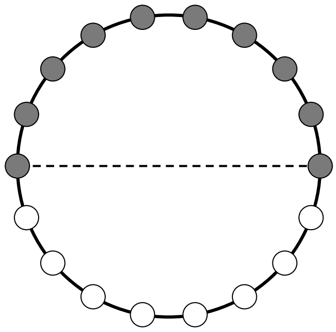
If colour does not matter, there are $3 \times 4 \times 3 = 36$ combinations. Of these 4 have two browns, 3 have two blues, 3 have two whites and 3 have two yellows, and none have all three the same. So there are $36 - 13 = 23$ combinations,

hence (A).

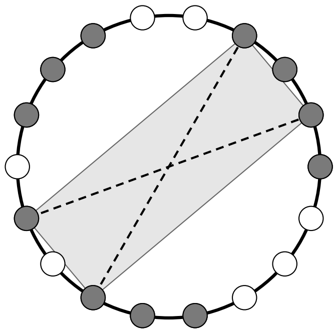
6. 2014 J27

Eighteen points are equally spaced on a circle, from which you will choose a certain number at random. How many do you need to choose to guarantee that you will have the four corners of at least one rectangle?

- The four corners of an inscribed rectangle appear as the ends of two diameters. It is possible to choose 10 points without having two complete diameters, as for example, the 10 consecutive points shaded below:



However, once 11 or more points are chosen, then at most 7 diameters are incomplete. So at least 2 diameters are complete, forming a rectangle.



Consequently 11 points are needed to guarantee one rectangle,

hence (11).