LESSON CARD

Magic Dice

An activity suitable for Australian years 2–6

AUSTRALIAN MATHS TRUST

Learning areas: Number and place value, patterns and algebra, linear and non-linear relationships, data representation and interpretation. Links to the applicable Australian Curriculum content descriptors are on page 4.

Resources: Lots of regular six-sided dice (plus 8-, 10-, 12- or 20-sided dice for later challenges). Visit www.amt.edu.au/resources-for-the-classroom for additional resources, including nets to create your own dice from paper or card, plus a summary table to help with Challenge (b).

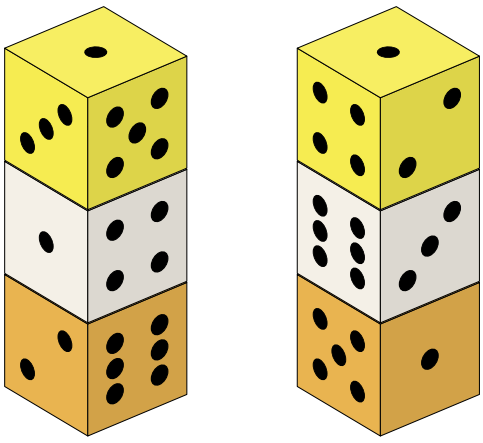
Magic Dice

In this activity you will learn a trick to amaze your family and friends!

Take three dice and stack them on your desk as shown. If you look from different angles, the following numbers are visible:

top layer:	1, 3, 5, 2 and 4
middle layer:	1, 4, 6 and 3
bottom layer:	2, 6, 5 and 1

So the hidden numbers are 6, 2, 5, 3 and 4. Adding these up, we see that the *hidden total* is 20.

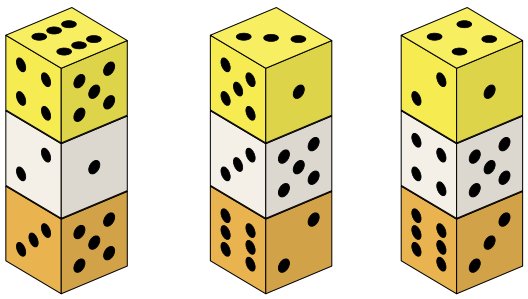


front view back view

Tip: When adding up the hidden total, start from the top and work down: '6 plus 2 equals 8, plus 5 equals 13, plus 3 equals 16, plus 4 equals 20.'

Challenges

(a) Create the three different stacks shown here. What is the hidden total for each one?



(b) Draw up a table to summarise your results so far, for example:

top face	visible side faces			hidden faces	hidden total
1	3,5,2,4	1,4,6,3	2,6,5,1	6,2,5,3,4	20
6	4,5,...	2,1,...			
3	5,1,...				
⋮	⋮	⋮	⋮	⋮	⋮

(c) Make your own stacks with three dice and add the results to the table. What do you notice? There is a quick way to work out the total without knowing all of the faces.

(d) Ask a classmate to build a stack of three dice. Quickly look at the stack and then announce the hidden total. Practice makes perfect!

(e) How is the trick affected when you use a different number of dice? Come up with your own rule and practise the trick with your classmates.

(f) Create a stack of dice where the hidden total is 31. How many dice do you need to use and what is the value of the top face?

(g) Ask a classmate to secretly build a stack of any height and get them to tell you the hidden total. Without looking, tell them the number of dice and the value of the top face. As always, practice makes perfect!

(h) Find out what happens when you use the following types of dice instead:

- 8-sided (octahedron)
- 10-sided (pentagonal trapezohedron)
- 12-sided (dodecahedron)
- 20-sided (icosahedron)

[Note: the answer to this may depend on how the manufacturers have designed the particular dice you are using.]

(i) There is one other common type of dice which is missing from the list above. Which is it and why?

Some answers

(a) $1+4+3+1+6=15$

$4+1+6+4+3=18$

$3+6+1+2+5=17$

(b)

top face	visible side faces			hidden faces	hidden total
1	3,5,2,4	1,4,6,3	2,6,5,1	6,2,5,3,4	20
6	4,5,3,2	2,1,5,6	3,5,4,2	1,4,3,1,6	15
3	5,1,2,6	3,4,4,2	6,2,1,5	4,1,6,4,3	18
4	2,1,5,6	4,5,3,2	6,3,1,4	3,6,1,2,5	17
⋮	⋮	⋮	⋮	⋮	⋮

(c) From the highlighted values in the table above, we notice that the value of the top face plus the hidden total always equals 21:

$$1+20=6+15=3+18=4+17=21.$$

This works because opposite faces on 6-sided dice always add to 7 (see also the comments in part (h)). So, to work out the hidden total, just subtract the top value from 21.

The tip on the first page suggests that you add the values from the top down. This tip helps to keep the illusion alive for your unsuspecting audience. If you add from the bottom up instead, then each running total will always involve the numbers 7 and 14, due to the opposite sides involved, and this might be enough to give away the secret!

(e) Removing or adding dice will decrease or increase the hidden total by 7 each time. So, for example, with four dice the hidden total is found by subtracting the value of the top face from 28. The general rule is

$$\text{hidden total} = 7 \times \text{number of dice} - \text{top face}.$$

(f) With four dice, hidden totals are less than 28. With five dice, hidden totals are less than 35. So to get a hidden total of 31 there must be five dice. The top face is 4, since $35-4=31$.

(g) Round the hidden total up to the next multiple of 7, say N. Dividing N by 7 tells you the number of dice. Subtracting the hidden total from N tells you the value of the top face.

(h) The trick relies heavily on the fact that opposite faces always add to the same value, which seems to be fairly universal for 6-sided dice. However, this is not always true for other types. If your dice do have this property, then the trick can be adapted. For example, given three 8-sided dice with opposite faces adding to 9, we have

$$\text{hidden total} = 27 - \text{top face}.$$

In general

$$\text{hidden total} = (1 + \text{number of faces}) \times \text{number of dice} - \text{top face}.$$

There is another complication: not all dice are necessarily numbered starting at 1. For example, it is quite common for some 10-sided dice to be labelled from 0 to 9 instead of 1 to 10. The trick can still be adapted, but care must be taken to know exactly how many dice of each type are being used in the stack.

Finally, there is no reason that all of the dice have to be the same type. All you need to know is the total of the opposite faces in each one.

(i) The 4-sided dice (tetrahedron) are missing from the list because they are impossible to stack!

For further hints and tips, contact mail@amt.edu.au.

Australian Curriculum content descriptors

The following is not intended to be an exhaustive list, but indicates how the above activity aligns with various stages of the mathematics curriculum. Follow the links to the ACARA website for elaborations.

- Year 2, ACMNA029 Explore the connection between addition and subtraction
- Year 3, ACMNA060 Describe, continue, and create number patterns resulting from performing addition or subtraction
- Year 4, ACMNA083 Find unknown quantities in number sentences involving addition and subtraction and identify equivalent number sentences involving addition and subtraction
- Year 5, ACMNA121 Find unknown quantities in number sentences involving multiplication and division and identify equivalent number sentences involving multiplication and division
- Year 6, ACMNA134 Explore the use of brackets and order of operations to write number sentences