LESSON CARD

Scaly Tiles

An activity suitable for Australian years 5–12

AUSTRALIAN MATHS TRUST

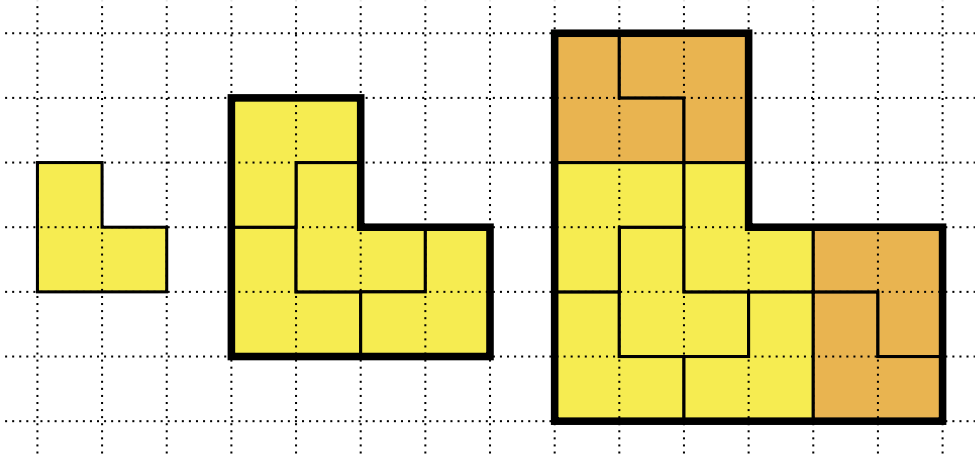
Learning areas: Shape, geometric reasoning, similarity, scale factor, transformations, patterns and algebra, surds, Pythagoras' theorem.

Resources: Square grid paper and isometric grid paper.

For the links to the applicable Australian Curriculum content descriptors, full solutions and additional resources, including downloadable templates visit www.amt.edu.au/resources-for-the-classroom.

Scaly Tiles

The L-shaped tile on the left is formed by joining three identical squares. This type of tile is called *2-scaly* because a number of them can be arranged, without gaps or overlaps, into an enlarged copy of itself which is twice as big (it has been scaled by a factor of 2). It is also *3-scaly*, because a number of them can be arranged into an enlarged copy which is 3 times as big.



original tile | **2-scaly** (2 × original size) | **3-scaly** (3 × original size)

Sometimes there is more than one way to arrange the tiles to make the enlarged copy. For example, in the 3-scaly arrangement on the right, either of the two dark rectangles could be tiled with small tiles in a different way.

Also note that when making the enlarged copy, the original tiles can be flipped over as well as rotated.

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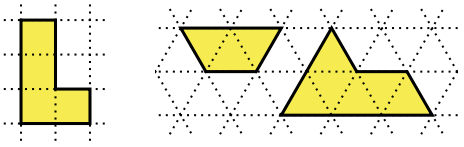
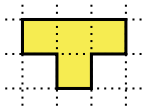
Challenges

(a) Show that each of these tiles is both 2-scaly and 3-scaly. Remember that tiles can be flipped over, as well as rotated.



(b) Show that this T-shaped tile is 4-scaly. Explain why it is *not* 2-scaly or 3-scaly.

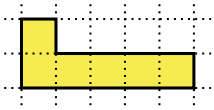


(c) How many **original** tiles fit into a 2-scaly arrangement? How many fit into a 3-scaly arrangement? What is the rule? Can you explain it?

(d) Explain why every 2-scaly tile is also 4-scaly. Show examples using the tiles in (a). For which other values of n is a 2-scaly tile also n-scaly?

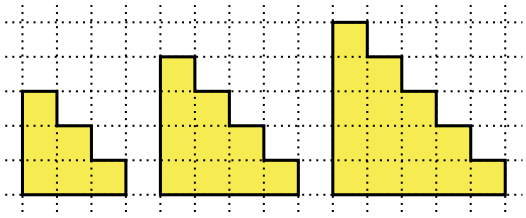
(e) Explain why every tile which is both 2-scaly and 3-scaly is also 6-scaly. Show examples using the tiles in (a). For which other values of n is a 2- and 3-scaly tile also n-scaly? What is the general rule?

(f) Find the smallest n for which this long L-shaped tile is n-scaly.

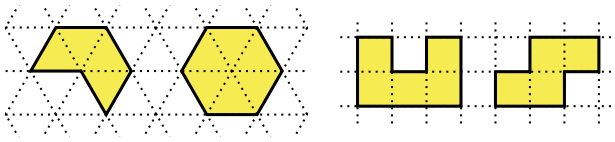


(g) Explain why every triangle is n-scaly for all n.

(h) A staircase tile can be made with any number of steps s, as shown. Explain why every staircase tile is n-scaly for some n, and find a rule for n in terms of s.

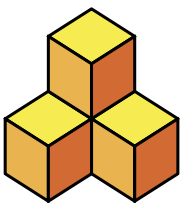


(i) Explain why none of these tiles is n-scaly for any n.



(j) Find an example of a triangle which is $\sqrt{2}$-scaly, one which is $\sqrt{5}$-scaly, and one which is $\sqrt{10}$-scaly. (Hint: by (c), how many tiles are needed?)

(k) This 3D block is made of four identical cubes. The visible cubes are all joined to one face of the hidden cube. Is this block 2-scaly? Is it 3-scaly? (Hint: how does (c) change?)



(l) Research how scaly tiles are related to the concepts of non-periodic tilings and self-similarity in fractals.