



2021 Maths Enrichment Director's Handbook



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AMT.
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ME.
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ENRICHMENT

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Information about Maths Enrichment

Instructions and information for students

Unlike the student problems in the Maths Challenge, the problems in Maths Enrichment are generally based on topics studied in the Student Notes, whether in Ramanujan, Newton, Dirichlet, Euler, Gauss, Noether or Pólya. In the Student Notes there are a number of examples and exercises with solutions for most topics, so that students have an opportunity to study similar questions to the given problems.

Make sure that the students understand the instructions given inside the Student Problems book. These are as follows:

Instructions for students

1. Before attempting the problems in this book, work through the appropriate sections of the Student Notes.
2. Solutions to the problems must be completely your own work. You may use resources such as text books or library books but *not seek help from other people, including people whom you could contact via the internet*. A calculator or computer may be used, but be sure that any programs used are fully and carefully explained.
3. Submit careful solutions to these problems according to the schedule provided by the Maths Enrichment Director at your school.
4. Your solution for each problem will be scored out of four marks. *Show your working and explain your reasoning for all problems even if this is not explicitly requested*. Marks will be awarded for clearly expressed arguments and careful reasoning. It is important to submit partial solutions to problems you have not completed as these may be worth some marks.

Further helpful information for students

- While there will be some problems where the only technique available is the ‘worry-it-to-death’ approach – testing an exhaustive (in both senses of the word!) number of cases – be on the lookout for neat short solutions. This will require the use of the mathematical ideas presented and some ingenuity.
- Additional help can be requested by contacting your teacher or school’s Enrichment Director for suitable resources or texts.
- A schedule for planning your chapter study and for submission of problems will be provided by your school’s Enrichment Director.
- Some problems may prove to be frustrating or too difficult. It is not necessary to submit all the problems; leave out some and do the later ones.

- We recommend that students compile an ongoing summary of the facts and techniques learnt in Enrichment. This will help with further studies in mathematics.
- The Student Notes will be useful in future secondary school and tertiary studies in mathematics and will be a valuable addition to your library.

How much may a teacher help?

As stated in the instructions to students, solutions to the problems must be completely their own work. However, if there are circumstances when students cannot proceed, perhaps because of language or terminology, then teachers may give some guidance. Further general problem-solving strategies may also be discussed.

The following general guidelines are designed to assist teachers in deciding what they may tell students.

- Emphasise the importance of reading the relevant parts of the text and solving the related problems.
- Help if asked, but only give as much help as will get the student thinking in the right direction.
- Give hints very sparingly and never in such a way that a solution is directly revealed.
- Give general problem-solving hints including:
 - Direct the student to consider a similar problem on a different scale.
 - Suggest where appropriate that the student make a table, list, diagram, etc.
 - Remind the student 'There are more options than you think. Can you look at it another way?' Don't show the other way(s) but rather help them to take a broader view and consider alternatives.
- Help the student to clarify their thinking by asking questions such as:
 - Do you understand the question?
 - What are you told in the question?
 - What are you required to find?
 - Are you using all the information?
 - Do you understand the terminology?
 - What have you found so far?
 - Can you simplify the problem?
 - How about writing something down?
- Assist with the terminology, particularly with students from years 4 to 8, and with students whose first language is not English. For example, words such as 'random' may need to be explained as may the means by which scientific notation is displayed on the calculator. A general discussion of exponents and scientific notation may be necessary.

- Encourage students to look for meanings of words and lead them to suitable references and resources in the library if necessary. Don't ignore the potential of a mathematical dictionary.
- Emphasize that marks will be awarded for reasoning and that arguments need to be written out fully and carefully.

Schedules and marking

Student schedules and regular reporting

Consider the time available in your school from the commencement (April - June) of the Maths Enrichment through to its finish in late September. There will be school examinations, holidays, excursions and so on. There are 8 problems in Ramanujan, Newton and Dirichlet, 12 problems in Euler and Gauss, and 16 problems in Noether and Pólya that the students will have to attempt during this period.

So realistically divide the time available to the student (and yourself) into a reasonable schedule. The recommended time period is 12-16 weeks, dependent on the level. It is also possible to break the program into parts. For example, set the first half for 6-8 weeks in term 2, and the second half for 6-8 weeks in term 3.

Note that such a schedule will mean that some students complete all the exercises in the Student Notes and then attempt the Enrichment problems, while other students will only do a few exercises before attempting the problems.

We recommend that you organise regular classes once or twice a week in order to discuss topics and exercises in the Student Notes and, later, the students' submitted solutions to the Enrichment problems.

Marking and recording results

We suggest that you mark papers on a regular basis (see page 9).

In designing the schedule, organise the deadlines for submission of Enrichment problems so that you can correct them quickly. For example, you may wish to set aside two free periods on say Tuesday afternoon for marking the problems, so make the deadline 9 am Tuesday for the students.

An electronic results spreadsheet and a link to an online questionnaire will be emailed to your Enrichment Director. Contact the AMT office at mcy@amt.edu.au if you have any concerns.

Maths Enrichment and the internet

It is becoming more common to find answers to Enrichment problems on the internet. Some students submit our original questions to sites from which they think they can get the answers. This is a source of continuing difficulty for the AMT. Where there is doubt about a student's answers, ask the student to explain their answers. Mark their answers accordingly.

Student success

The primary purpose of the Enrichment program is for students to work their way through a systematic course in problem solving. This will position them to be able to attempt some other significant problem-solving competitions such as the AIMO and gain confidence. Students may encounter mathematical ideas they do not immediately understand. Explain that not all students are expected to complete all the work in the scheduled time.

About Maths Enrichment

Ramanujan, Newton, Dirichlet, Euler, Gauss, Noether and Pólya all include Student Notes, Student Problems book and Teacher Guide. Each stage introduces some mathematics not usually taught in school; each stage is independent of the others until Noether which builds on Gauss.

Encourage students to work through the material of each chapter of the Student Notes attempting solutions to the exercises as they go, initially without recourse to the solutions provided in the Notes. Students can check and compare their solutions with the given solution, or, if a solution is not found, use the solution to learn techniques that may be able to be applied to another problem.

All contain miscellaneous problems designed to reinforce the knowledge and skills of students and to broaden the students' experience of solving mathematical problems.

We hope that as students work through this material they will add to their set of mathematical tools and improve their problem-solving skills. Many of the exercises require a non-standard use of the tools while some simply reinforce a concept which has been introduced. The students are encouraged to look for 'neat' solutions rather than achieve the result through a lengthy or awkward approach. At the same time, persistence is often required to solve the problems and is a quality to be fostered in all young problem solvers.

Ramanujan (updated in 2020)

This has eight chapters and is designed for mathematically able year 4 and 5 students. It is also most appropriate for use with year 6 and 7 students. The topics and techniques covered are:

- Fast arithmetic
- Special numbers
- Counting techniques
- Polyominoes
- Making a list
- Guess and check
- Using a diagram

The Student Problems book has 8 problems.

Newton (updated in 2021)

This has eight chapters and is designed for mathematically able year 5 and 6 students. It is also most appropriate for use with year 7 and 8 students. The topics covered are:

- Estimation
- Polyhedra
- Divisibility
- Patterns
- Colouring problems
- Arithmetic in other bases
- Problem-solving techniques

The Student Problems book has 8 problems.

Dirichlet (updating in 2022)

This is designed for students in year 6 or 7. Three of the eight chapters revolve around a story which illustrates some problem-solving techniques: using logic, solving a simpler problem, and working backwards. The other five chapters cover:

- Tessellations
- One-handed arithmetic
- Time, distance, speed
- Working with patterns
- Recurring decimals

The Student Problems book has 8 problems.

Euler

This is designed for interested and talented students in year 7 and 8. The topics considered in this series are:

- Primes and composites
- Least common multiple
- Highest common factor
- Arithmetic sequences
- Figurate numbers
- Congruences
- Properties of angles
- Counting techniques
- Pigeonhole principle

The Student Problems book has 12 problems.

Gauss

This is designed for talented students in year 8 and 9. It introduces the use of computer spreadsheets. The topics covered include:

- Parallels
- Similarity
- Pythagoras' theorem
- Spreadsheets
- Diophantine equations
- Counting techniques
- Congruence

The Student Problems book has 12 problems.

Noether

This is designed for talented students in year 9 and 10, following on from Gauss. The topics considered are:

- Expansion and factorisation
- Inequalities
- Sequences and series
- Number bases
- Methods of proof
- Congruence
- Circles
- Tangents

The Student Problems book has 16 problems.

Pólya

This is specifically designed for the top 5–10% of year 10 students. There are two problems on each of the following topics:

- Functions
- Symmetric polynomials
- Geometry
- Inequalities
- Functional equations
- Number theory
- Counting
- Graph theory

The Student Problems book has 16 problems.

It would be useful for students to compile an ongoing summary of the facts and techniques learned and use them to obtain their own solutions to the examples and exercises.

Administrative procedures and guidelines

Maths Enrichment package

1. The Maths Enrichment package includes the following:

a. Director's materials

- Letter to the school's Maths Enrichment Director
- Maths Enrichment Director's Handbook

b. Maths Enrichment materials

Each student registered will receive

- 1 x Student Notes
- 1 x Student Problems

The Maths Enrichment Director will receive a Teacher Set/s (for every stage in which students are participating) containing

- 1 x Teacher Guide
- 1 x Student Notes
- 1 x Student Problems

NB 1 x Teacher Set for 20 entries or fewer

(1 extra Teacher Set for 21–40 entries)

(1 extra Teacher Set per multiple of 20 entries)

2. A results spreadsheet will be sent to the Enrichment Director. If there are any concerns contact mcya@amt.edu.au

We suggest that the Maths Enrichment package be placed in a secure place until ready to use.

Director's Handbook label

Explanation of Coding

Your AMT school code and school name

Student Bks:

RS: 0, NTS: 0, DS: 12, ES: 0, GS: 0, NS: 0, PS: 0

Teacher Sets:

RT: 0, NTT: 0, DT: 1, ET: 0, GT: 0, NT: 0, PT: 0

Student Bks: number of Student Problems books and Student Notes for registered students, where RS# = Ramanujan, NTS# = Newton, DS# = Dirichlet, ES# = Euler, GS# = Gauss, NS# = Noether, PS# = Pólya.

Teacher Sets: 1 x Complimentary Teacher Set for every 20 students (or part thereof) entered, where RT# = Ramanujan, NTT# = Newton, DT# = Dirichlet, ET# = Euler, GT# = Gauss, NT# = Noether, PT# = Pólya.

The day that students are given their Maths Enrichment materials

1. Collect the Maths Enrichment package from the secure place.
2. Collect the information below for each student and record on your spreadsheet:
 - a. their last name and first name
 - b. school year, sex, date of birth
 - c. population data questions
 - d. Enrichment stage entered (e.g. Ramanujan).

NB The results spreadsheet and a link to an online questionnaire will be sent to the school email contact.
3. Distribute to each participating student the appropriate set of two books, 1 x Student Notes and 1 x Student Problems. Please note that these books should be retained as the property of the student.
4. Make sure that participants understand the advice noted in their books and discuss the further information given on pages 1-2.
5. Prepare students' schedule for work on the Maths Enrichment.
 - a. Consider the time available in your school for the Enrichment program (see 'Student schedules and regular reporting' on page 3). There are up to 16 problems that the students can attempt.
 - b. Give the students reasonable short-term goals to complete various sections of work in the Student Notes. Students with differing abilities or other demands may be given different schedules. Thus schedule the time realistically by setting reasonable deadlines for the submission of each problem for each student.
 - c. Be sure that corrected work is not returned to any student until all students have submitted their attempts.
 - d. Explain to the students that the scope of the set work will be demanding for most to finish in the scheduled time, but in completing some of it, they will be achieving much.

Regular marking of students' attempts at problems

Marking and scoring procedures are given in the Teacher Guide, as follows.

Instructions for Teachers

Mark papers on a regular basis

1. Award marks as shown in the marking scheme in this book.
2. If an incomplete attempt is presented which is different from the solution supplied, award marks for comparable work. To assist you to do this we have summarised the work for which each mark is awarded and, in some cases identified some alternative solutions. If you are not sure what mark to award, give the student the benefit of the doubt. Awarding a 0 will ensure the student will receive a certificate.
3. A Participation certificate will not be awarded to a student if all questions for that student are marked 'NOT ATTEMPTED'.

Record results on a regular basis

For each student's attempt at each problem, record results on the spreadsheet supplied.

Discuss corrected work on a regular basis

It is important to discuss the solutions with the students as soon as possible after the questions have been attempted by all participating students and marked.

Before 14 October 2021

Return the results spreadsheet by email to mcya@amt.edu.au and complete the online questionnaire.

Ensure that both the results and questionnaire reach the AMT office by 14 October 2021.

1. On completion of marking, record each student's score in the appropriate place on the results spreadsheet making sure that the correct stage (Ramanujan, Newton, Dirichlet, Euler, Gauss, Noether or Pólya) has been marked.
2. Give out the solutions to the relevant problems and discuss the scores with the students.
3. Please complete the online questionnaire. Your comments, suggestions and guidance about the Maths Enrichment are invaluable to improve procedures and concepts of this important initiative of the Australian Mathematical Olympiad Committee.

4. Return the completed Maths Enrichment results spreadsheet by email to mcy@amt.edu.au by **14 October 2021**.
5. Schools can expect to receive results and certificates for the Maths Enrichment by mid-November.

October – December 2021

Follow-up support to students

It is expected that some of the topics in the various books of Ramanujan, Newton, Dirichlet, Euler, Gauss, Noether and Pólya may not have been completed by many students. Why not encourage them to work on the material during the last couple of months of the year?

Australian Intermediate Mathematics Olympiad (AIMO)

Thursday 9 September 2021

Year 7 to 10 students who do well or show commitment in the Maths Challenge and/or Maths Enrichment should be encouraged to enter the Australian Intermediate Mathematics Olympiad (AIMO). The AIMO is a four-hour closed book contest, consisting of ten questions based on broad mathematical ideas. Students sit the AIMO within their own schools, and papers are sent to AMOC State Directors for marking.

Every participant receives a certificate and book prizes are awarded to students who are outstanding on a national basis.

Sample papers and solutions can be found on the AMT website: www.amt.edu.au/department/past-papers

AMOC Senior Contest

Tuesday 17 August 2021

Entry for the AMOC Senior Contest is by invitation only. For enquiries, please contact your AMOC State Director.

AMOC Senior Contest materials will be sent separately to those schools with invited students.

AMOC State Directors' contact details

ACT	Dr Chris Wetherell Email: chris.wetherell@amt.edu.au	02 6201 5330 (w)
VIC	Dr Philip Swedosh Email: pswedosh@gmail.com	0468 840 646 (mob)
NSW	Dr Dzmitry Badziahin Email: dzmitry.badziahin@sydney.edu.au	02 9351 3279 (w)
SA	Mr Michael Bammann Email: mbammann@gmail.com	0404 008 697 (mob)
NT	Dr Ian Roberts Email: ian.roberts@cdu.edu.au	0403 406 999 (mob)
TAS	Dr Kumudini Dharmadasa Email: kumudini@utas.edu.au	03 6226 2491 (w) 0405 360 734 (mob)
QLD	Dr Brenton Gray Email: brenton@wildfiresports.com.au	0422 248 549 (mob)
WA	Dr Greg Gamble Email: greg.gamble@uwa.edu.au	08 9266 3482 (w)

If unable to contact the above please try:

Australian Maths Trust Email: mcy@amt.edu.au	02 6201 5136
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Notes

2021 DIRECTOR'S HANDBOOK