Maths Enrichment

Newton Student Notes

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AUSTRALIAN MATHS TRUST
MATHS ENRICHMENT

Contents

Chapter 1
Penguin Estimation

Sliding down the icy slopes was so much fun for the feathered friends. Sliding on her tummy, Mali, always the bravest penguin of the four, was first to reach the bottom of the icy hill.

Levi and Telia both slid down, always eager to beat the other. Squirt tumbled head first and came crashing into the other three who had stopped in front of a massive building.

Squirt got up, flattened his messed up feathers and joined the others staring in amazement.

Finally they had reached the Mawson station. Led by their curiosity and mischievous nature, the friends had travelled many kilometres to find the station to learn more about science.

'Quick, everyone, follow me.' Mali led the way. With their backs against the wall and taking sideward steps, together they moved around the building until they came to a door.

With quick beak movements left and right to check no one was watching, the friends snuck into the station, avoiding being seen by the scientists. Finally they arrived at an empty room.

They found themselves in front of a large screen. A film showed large groups of penguins marching over the ice against ferocious winds.

'This is amazing,' said Mali. 'I'm sure that is my mum there.'

'I don't know—she looks the same as my mum,' exclaimed Squirt.

'Where are they marching to?' asked Levi.

'This is the trek our mums took from the colony site, after we were hatched. They marched 112 kilometres to the ocean shore to collect food. We were slowly hatching in our eggs while our dads watched over us,' explained Mali.

'Wow! How long did that 112-kilometre trek take? That's a huge distance. I think it would take me a year to get there,' said Levi.

'Five days. I think I could do it in five days!' remarked a confident Squirt.

'That's impossible Squirt. Go on, show us how you can do it.'

'Hmm, well . . . perhaps . . . ' Squirt replied, now slightly embarrassed.

Chapter 2

Polyhedra

Opening problem

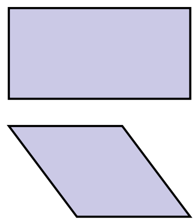
> A solid shape is made of 12 black pentagons and 20 white hexagons, like a soccer ball. How many edges and corners does the solid have?

You could get hold of a soccer ball and try to count the corners and edges one by one, but read on to learn more about this type of problem!

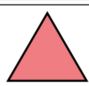
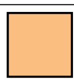
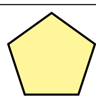
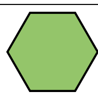
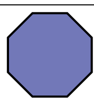
Polyhedra and Platonic solids

Plato was a Greek scientist, philosopher and mathematician who lived from 427 to 347 BCE. He founded an 'academy' in about 380 BCE and investigated these solids.

A *polygon* is a planar (flat) closed shape with straight sides. A *regular* polygon is one with all of its interior angles equal and all of its sides equal in length. So the polygons on the right are not regular—the first because not all the sides are equal and the second because not all the interior angles are equal.



The properties of some common regular polygons are in the table shown.

Regular polygon	Picture	Number of sides	Size of interior angle
Equilateral triangle		3	60°
Square		4	90°
Pentagon		5	108°
Hexagon		6	120°
Octagon		8	135°

Chapter 3
Divisibility

Opening problem

Without using a calculator, find a number between 35000 and 36000 that leaves no remainder when divided by 5, 8, 9 or 11.

You could check each number one at a time by hand . . . but read ahead for some useful shortcuts!

Factors and multiples

In this chapter we will only be dealing with the the counting numbers, also known as the *natural numbers*: 1, 2, 3, ... A number is a *factor* of another number if it divides into that number exactly. So 7 is a factor of 56, since $56 \div 7 = 8$. Looking at it the other way round, 56 is a *multiple* of 7 because $56 = 7 \times 8$. It is also a multiple of 2, 4, 8, 14 and 28. *Check this for yourself.*

Instead of saying '7 is a factor of 56' or '56 is a multiple of 7', we can also say '7 *divides* 56' or '56 *is divisible by* 7'.

We are interested in finding ways to decide whether one number is divisible by another without actually doing the division. That is, we want to find tests for divisibility.

For example, is the number 1 234 567 890 987 654 321 divisible by 3? Remember that 'divisible by 3', means 'divides exactly by 3 without remainder'. But doing the division in this case would be very time consuming. We will come back to this problem soon.

But first, let's begin with some easier tests.

Tests for divisibility by 2, 5 and 10

We have a special name for numbers which are divisible by 2: they are the *even* numbers. So everyone should already know a test that tells us whether a number is divisible by 2, just by looking at the digits.

Write down the test in your own words.

Chapter 4

Problem Penguins

The penguin quartet continued to explore the centre.

'Hey guys, we've lost Mali,' noticed Telia. At each door, the other three would stop and take a quick penguin peek into the room to scan the situation and report on their missing-in-action friend.

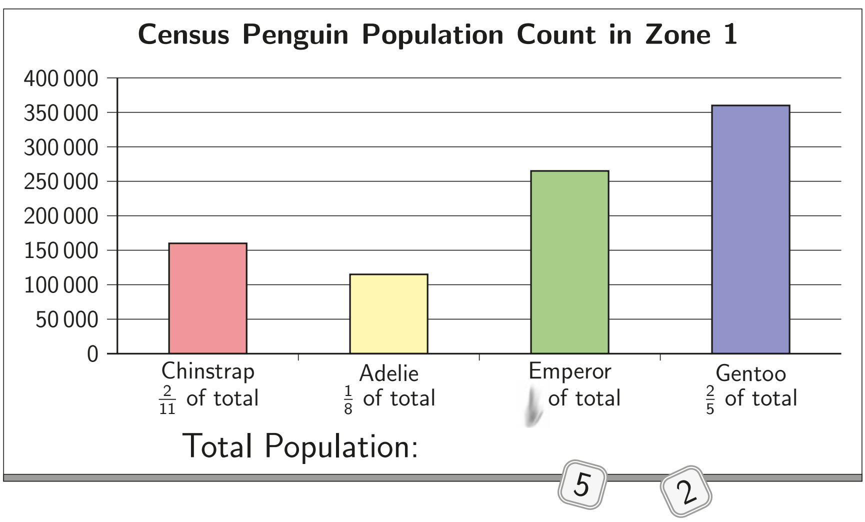
Suddenly they heard Squirt cry out in joy: 'Missing Mali located!' They joined Mali standing in front of a large whiteboard.

Telia read out the title at the top of the whiteboard: 'Census Penguin Population Count in Zone 1'. 'This is great, I've always wanted to know how many emperor penguins, just like us, there are.'

Levi read 'Total population', which was written on the board next to a 6-digit number. He explained, 'There are tiles on the board ledge, each with a single digit on them forming a 6-digit number. This represents the total population.'

Just as Mali was about to read the number out, inquisitive Squirt climbed onto a table and jumped onto the top of the whiteboard. Losing his balance, as usual, the whiteboard started to tilt backwards and Squirt slid down the front side of the board. Scrambling to save their friend, the other three quickly joined flippers to help soften the fall and let Squirt land safely.

'Crisis averted again,' said Telia. 'We saved you, but your feathers have wiped off part of the board and knocked the digits that formed the total population off the ledge.' Squirt rubbed his sore head, concerned about what had happened. He looked at the whiteboard.

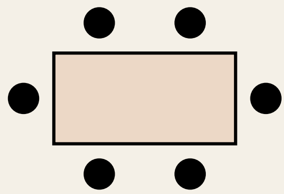


Chapter 5

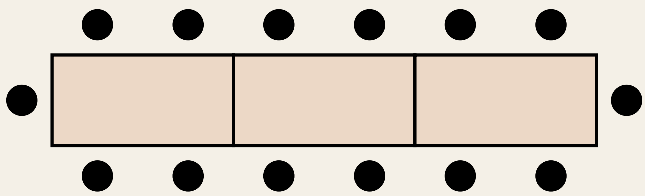
Recognising and Describing Patterns

Opening problem

A table used by a caterer can fit 6 people around it like this:



Two or more tables can be joined together to seat more people, like this:



If the caterer puts 50 tables together in this way, how many people could be seated? What if the caterer puts the 50 tables together with their long sides touching instead of their short sides?

It wouldn't take you *too* long to draw a diagram with 50 tables . . . but often the key to solving a problem is understanding the pattern.

Sequences

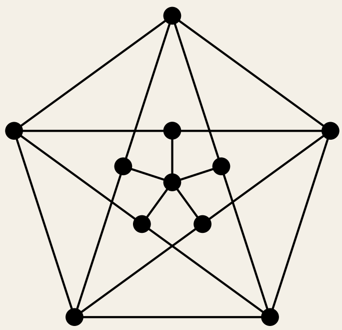
In this chapter we look at patterns of objects, called *sequences*, and investigate the rule that tells us what the next object in each pattern will be. The members of a sequence are called *terms*. We describe each term them by their position in the sequence. For example, 'term 3' is a shorthand way of saying 'the third object in the sequence'.

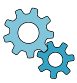
Often the terms of a sequence will involve whole numbers. It is very useful to find a rule which tells us the value of each term based on its position, if possible, because then we can use the rule to make predictions about the pattern.

Chapter 6
Colouring Graphs

Opening problem

Assign one colour to each dot so that no two dots joined by a line have the same colour. How many colours do you need?





In this chapter you will see why this is an important type of problem and how to be sure you have the right answer.

Vertex Colouring

The girls in the new Riverton Girl Guides unit come from the surrounding area, but not many of them know each other. Their leader Jill wants to divide them into groups for a number of activities. In order to mix them as much as possible, no two girls that know each other are allowed in the same group. What is the smallest number of groups that Jill can arrange?

Of course, we can't answer this question without knowing which girls know each other. We could list for each girl the names of all the other girls she knows and then try to work from those lists, but this can get very fiddly. Another approach is to use a *graph*.

A graph is a collection of dots, called *vertices*, joined by lines or curves, called *edges*. In Chapter 2, you already saw that 'vertices' and 'edges' are important features of a polyhedron (a solid with flat faces). In fact, a polyhedron *is* a special example of a graph. But in general, we do not need the edges to be

Chapter 7
Penguins – A Class Act

The penguins were quite intrigued with what they saw in the different rooms. In one room there were measuring devices for temperature, weight and distance. Next door there was a room filled with computers, followed by a room filled with video and audio recordings. Then there was a labyrinth of laboratories and a massive room filled with hiking equipment, tents and extra warm clothing.

Surprisingly, they came across a section of the station that had an ice floor with a large hole in it connected to the ocean. They also found rooms where the scientists grew their own vegetables using hydroponics and stored lots of food supplies.

Wandering around the lettuce trays Telia noticed a sign:

$$\textbf{Fertiliser : Water} = \textbf{3 : 47}$$

'I've never seen a colon (:) between two numbers before. What does it mean?' asked Telia.

'Neither have I,' added Levi.

'That is a ratio,' explained Mali. 'In this case it means, for example, if you use 3 cups of fertiliser you will need to add 47 cups of water. Or if you wanted to make a small mixture, say 50 millilitres, you would measure 3 millilitres of fertiliser and 47 millilitres of water to make up the 50 millilitres.'

'I think I have it,' said Levi.

'Sometimes it helps to understand it if you act it out. First we need to go and catch some food, then we can act on using ratios.'

The penguins quickly waddled to the room that had the ice floor.

'Okay, I've got an idea. We are going to catch krill and share them between us using ratios. Squirt and Telia, you are both quick, so Squirt I need to you to catch 40 krill, Telia please catch 38, Levi 12 and I will catch 6. Off we go!' instructed Mali.

Within moments the friends had made a quick dive into the ocean beneath the ice, caught the right number of krill and jumped back up onto the ice floor.

Chapter 8

One-handed Arithmetic

Opening problem

On the planet Octa, the people have just four digits on each hand, so they count using an octal (rather than a decimal) system. This means they only have digits 0 to 7, so when they get to eight they need to use the next column. How would the Octal people write the number 1000?

You could try listing all one thousand numbers and see where you end up ... is there a quicker way to solve this type of problem?

The decimal system

The number system we use is called the *decimal* system. This means it uses base 10.

We have ten symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9, which are called digits. When we want to go to the next number past 9, instead of having a new symbol, we combine the ten 1s or units into one group and call it 1 in the next column to the left (the tens column) and start the units off again: 10, 11, 12, 13, ..., 19, 20, 21, ... If we keep counting on, we will come to 99.

Adding 1 more makes ten 10s or one hundred. We put 1 in the next column to the left (the hundreds column), and the units start off again. Every time we use up the ten digits in one column, we need another column to the left. This happens at 10, $100 = 10 \times 10 = 10^2$, $1000 = 10 \times 10 \times 10 = 10^3$, $10000 = 10^4$, and so on.

We can break down a number into units, tens, hundreds, etc. Thus 43 means $\mathbf{4} \times 10 + \mathbf{3} \times 1$, and 2735 means $\mathbf{2} \times 1000 + \mathbf{7} \times 100 + \mathbf{3} \times 10 + \mathbf{5} \times 1$.

The base 10 system probably developed because early counting was done using the fingers of both hands, and we have ten fingers.

Two illustrations of how the decimal or base 10 system works are shown.