Maths Enrichment

Ramanujan Student Notes

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AUSTRALIAN MATHS TRUST
MATHS ENRICHMENT

Contents

Chapter 1
The Castle of Problems

They passed it every day on their way to and from school. Its enormous walls, with many windows at odd levels, kept them curious. The light that always appeared in the castle tower, day and night, kept them wondering. The howling of the wind around the castle filled the street with mystery.

It was midafternoon. Jasper and Anastasi had been dismissed early from their school sports carnival.

‘Anastasi, I see a door open around the side of the old castle. This is our chance to look inside. Come on,’ whispered Jasper as she crept through the garden and moved into the castle. They were barely inside the door when it slammed shut behind them. They turned quickly, but from the inside . . . there was no sign of a door!

‘Jasper, what have we done? We may never find our way out of here!’ cried Anastasi.

‘This is a problem! No one knows where we are. How will they ever think to look for us here?’ said Jasper in despair.

At that moment Anastasi noticed letters waving around in a 3D hieroglyphic image on the wall.

.SMELBORP FO ELTSAC EHT FO TUO
YAW RUOY DNIF LLIW UOY DNA
OG UOY SA SMELBORP EHT EVLOS
.TI OT REWSNA EHT WONK UOY ECNO
MELBORP A REGNOL ON SI MELBORP A

‘Smelborp?’ questioned Jasper. ‘This makes no sense! Could it be another language?’

Chapter 2

Arithmetrix

Opening problem

Without a calculator, find the missing digit:

$$195\,623 \times 75328 = 14\,73\mathbf{?}\,889\,344.$$

You *could* try multiplying out the numbers by hand . . . but read on to learn some handy arithmetic tricks!

Seeing the 9 times table on your fingers

The ability to do arithmetic accurately and quickly is based on a sound knowledge of tables. There are many patterns in the multiplication tables which help us to remember them but can also help us to do multiplications outside the usual tables.

You know the 9 times table of course:

$$\begin{aligned}
1 \times 9 &= 09\\
2 \times 9 &= 18\\
3 \times 9 &= 27\\
4 \times 9 &= 36\\
5 \times 9 &= 45\\
6 \times 9 &= 54\\
7 \times 9 &= 63\\
8 \times 9 &= 72\\
9 \times 9 &= 81\\
10 \times 9 &= 90
\end{aligned}$$

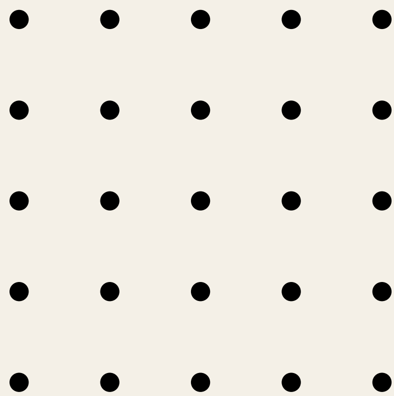
Write down any pattern you can see in the answers, then try to relate your pattern to the number that is multiplied by 9.

Chapter 3

Counting Techniques

Opening problem

How many different straight lines can you draw which pass through three or more of the dots in this 5×5 grid?



A systematic counting strategy is definitely needed for this kind of problem!

So you think you can count!

I expect by now you think you can count.

However, many problems involve careful counting of shapes, lines or other objects and students often find these very difficult. The trick is to come up with a careful strategy that makes sure you don't miss anything out, or count anything twice.

In this chapter, we look at a few tricks you can use to count things without making mistakes.

Chapter 4

In the Gauss Room

'Hope this is right,' said Jasper as she pushed open the door at the top of the staircase and they entered. The door closed slowly behind them. They looked around the Gauss room. Three walls were painted and one was bricked. Four labelled doors surrounded them. None of them had a handle and they could not be opened, other than by following the right set of instructions which had been painted on the walls.

'By our plan we now need to get to the Noether room,' said Jasper, as she walked over to read the instructions.

> TO OPEN THE DOOR LEADING
> TO THE NOETHER ROOM
> FOLLOW THESE INSTRUCTIONS:
>
> You need to press three bricks in the opposite wall. Starting from the top-left corner, the bricks are numbered 1 to 72, moving left to right. There are 12 bricks in each row.
>
> The numbers on the first and second brick you need to press have a difference of 2. The numbers on all three bricks are prime numbers. The third brick shares an edge with the second brick. The product of the numbers on the three bricks ends in 7.
>
> Press the correct bricks and the door will open.

'Where do we start, Anastasi?'

Chapter 5
Some Special Numbers

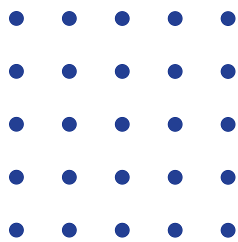
Opening problem

What is the sum of all the numbers from 101 to 200?

You *could* just add up all the numbers on a calculator ... but is there a faster way? Read on!

Square numbers

A square number is the result of multiplying a number by itself. For example, 25 is a square number since it equals 5×5. We write $25 = 5^2$ and say '25 is five squared'. The name 'square' is used because we can arrange that number of dots into a square pattern, like so:



The first five square numbers are 1, 4, 9, 16 and 25.

Exercises

1. Write down the next 5 square numbers.

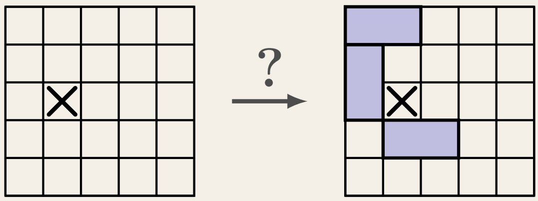
2. The first cube number is $1^3 = 1 \times 1 \times 1 = 1$. Write down the next 5 cube numbers.

3. What are the first two cube numbers which are also square numbers?

Chapter 6
Polyominoes

Opening problem

One square of a 5×5 grid is crossed out, as shown. Is it possible to cover the rest of the grid with 2×1 rectangles without any of them overlapping?

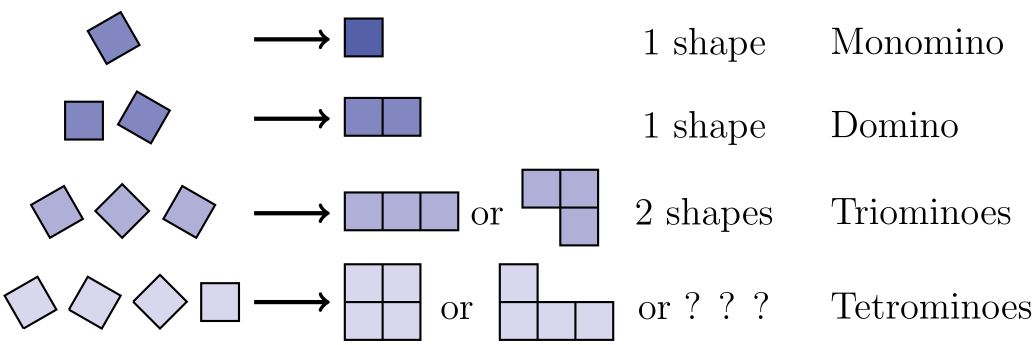


What if the crossed-out square is somewhere else?

It would take a long time to try all the possibilities. What if some were missed? Might there be a better way?

Shapes from squares: the Polyomino family

Polyominoes are shapes made from identical unit squares, joined edge to edge. You may have already seen this idea in Chapter 3.



There are more tetrominoes. See Exercise 1.

Chapter 7

In the Fibonacci Room

'I suppose this is the Fibonacci room, seeing there is only one door out of the Noether room and that was supposed to lead to the Fibonacci room, according to our map,' said Jasper as they entered yet another room in the Castle of Problems.

She pushed the door open at the end of the staircase that was full of dust and cobwebs. As they entered, they saw a large knight in shining armour standing still in the corner. Approaching the knight, they heard a voice booming out of the helmet.

'THE KING OF THIS CASTLE WAS VISITED BY KNIGHTS SUCH AS MYSELF AND BY PRINCES. EVERY KNIGHT BROUGHT GIFTS OF ONE GOLD AND TWO SILVER CHESTS OF JEWELS. EVERY PRINCE BROUGHT GIFTS OF THREE GOLD AND ONE SILVER CHEST OF JEWELS. IN TOTAL THE KING RECEIVED 34 GOLD CHESTS AND 33 SILVER ONES.

'YOU HAVE FIVE CHANCES TO CORRECTLY GUESS THE NUMBER OF KNIGHTS AND PRINCES THAT VISITED THE KING. IF YOU HAVE NOT FOUND THE CORRECT NUMBER OF EACH BY THE FIFTH GUESS, THE DOOR WILL NEVER OPEN.'

'Ten knights and 10 princes!' cried Anastasi, keen to give an answer.

'TEN KNIGHTS AND TEN PRINCES WOULD HAVE BROUGHT 40 GOLD AND 30 SILVER CHESTS. THIS DOES NOT MATCH THE NUMBER RECEIVED BY THE KING. YOU HAVE FOUR GUESSES LEFT,' replied the knight.

'Five knights and 20 princes,' gasped Jasper.

'THEY WOULD HAVE BROUGHT A TOTAL OF 65 GOLD AND 30 SILVER CHESTS. THIS DOES NOT MATCH THE NUMBER RECEIVED BY THE KING. YOU NOW HAVE THREE GUESSES LEFT!'

Chapter 8

More Special Numbers

Opening problem

Can you find an even number between 200 and 300 which has an odd number of factors?

You *could* list the factors of all the even numbers one at a time ... read ahead to discover a more efficient method!

Multiples and factors

Because $3 \times 5 = 15$, we say that 15 is a *multiple* of 3. It is also a multiple of 1, 5 and 15. 15 is the *product* of 3 and 5.

We can also say that 3 is a *factor* of 15. The other factors of 15 are 1, 5 and 15. Every factor of a number will divide into the number with no remainder.

Exercises

1. Write down the first 6 multiples of 7.

2. Write down all the factors of

(a) 18 (b) 24 (c) 36

Prime numbers

In Chapter 4, Anastasi and Jasper used their knowledge of prime numbers to escape the Gauss room in the Castle of Problems.

A *prime number* (or just *prime*) is a number greater than 1 which has only two factors: itself and 1. So 2, 3, 5, 7 and 11 are primes, but 1, 4, 6, 8, 9 and 10 are not.