



2022 Maths Challenge

Director's Handbook

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From the Director

Welcome to the 2022 Maths Challenge and congratulations on deciding to take the time and tantalising risk in being part of it. Whether this is your first time or 32nd time in the Challenge, we hope that you and your students find the experience fresh, enjoyable, and rewarding. It is worth noting the Challenge aims, which we all share and contribute to:

- encouraging and fostering
 - a greater interest in and awareness of the power of mathematics
 - a desire to succeed in solving interesting mathematical problems
 - the discovery of the joy of solving problems in mathematics
- identifying talented young Australians, recognising their achievements nationally and providing support that will enable them to reach their own levels of excellence
- providing teachers with
 - interesting and accessible problems and solutions as well as detailed and motivating teaching discussion and extension materials
 - comprehensive Australia-wide statistics of students' achievements in the Challenge.

Timing events such as the Challenge is important for it to be effective, but not always easy. We trust that amongst the constraints of busy school schedules, directors can find *four* suitable weeks (an increase from three weeks in previous years to allow more flexibility) in the Challenge allocated period from the date of delivery to 24 June 2022. Here are some time-saving suggestions which could help.

- Using the electronic sample letter to parents which is available by email from mcya@amt.edu.au.
- Recruiting colleagues to help with marking by allocating one problem to each person.
- Encouraging each student to use a separate bound exercise book for all their Challenge work, showing their preliminary investigations, progressive drafts, and final complete solutions.
- Recruiting parents, trainee teachers, or retirees with relevant experience to coordinate one or more levels of the Challenge for your school.
- Ensuring students know the difference between a simple answer or calculation and a full solution with reasoned argument.
- Encouraging feeder primary and other schools to participate in the Challenge so there is a wider pool of experience and support amongst local colleagues.
- Setting early deadlines for some problems so they can be marked while students work on others.
- Explaining to students any unfamiliar terms in a problem and clarifying a question where necessary.

Marking is always a quandary. The Problems Committee endeavours to present problems, marking schemes, and model solutions so they indicate as precisely as possible how marks should be allocated. Nevertheless, discretion is often required particularly for reasoning and explanation. Where there is only one mark available, the desire to award a half mark is strong; the stipulation not to do so is seen as unfair. However, students have four weeks to prepare and revise their solutions so

they are correct and complete. Past Challenge problems and solutions can help them see what is expected. Encourage students to criticise their own writing, to look at their solution as building a bridge of understanding. Half a bridge will never do but where there is doubt, award the mark.

I would like to record my thanks and appreciation to the members of the Challenge Committee for the time, expertise and enthusiasm they contribute to the Maths Challenge. Their names are recorded in the Teacher Guide. Collectively they have given over 370 years of invaluable service. My thanks go also to the many moderators for their insightful and meticulous review of Challenge drafts and to the dedicated and ever reliable Australian Maths Trust staff for the efficient administration of the Challenge program. We have in the Challenge a wonderful legacy of mathematics enrichment for young Australians, a program I believe that is unique in the world.

I wish you and your students well for Challenge 2022.

Kevin McAvaney
MYA Director

Administrative Procedures and Guidelines

A. On receipt of the Challenge package

1. You should have:

- Maths Challenge Director's Handbook
- Sealed package (to be opened at commencement of the Challenge) containing:
 - Student Problems books (Middle Primary, Upper Primary, Junior and/or Intermediate) as ordered
- Sealed complimentary Teacher set(s) containing:
 - Teacher Guide with solutions and marking schemes for the Challenge problems, as well as extension material
 - One each of the Middle Primary, Upper Primary, Junior and Intermediate Problems books.

IMPORTANT: You will receive the results spreadsheet via email. If you do not, please contact us at mcya@amt.edu.au to request it.

2. Explanation of the coding on the label attached to the front of this handbook:

Your AMT school code	(See below for explanation)
School name	MP:1 UP:0 J:2 I:0 T:1
The Maths Challenge Director	
.....	
.....	

where MP# reflects the number of Middle Primary Problems books (i.e. the number of registered Middle Primary students)

where UP# reflects the number of Upper Primary Problems books (i.e. the number of registered Upper Primary students)

where J# reflects the number of Junior Problems books (i.e. the number of registered Junior students)

where I# reflects the number of Intermediate Problems books (i.e. the number of registered Intermediate students)

where T# reflects the number of complimentary Teacher sets (one for up to 30 student entries and multiples thereof; if you have 31 students entered at any level, 2 complimentary Teacher sets will be included)

3. The Challenge Committee strongly recommends to schools that they formally allocate, if possible, special class periods on the school's timetable. This recommendation is based on the comments and suggestions of many teachers who reported most positively of the benefits to their students of similar arrangements made in previous Challenge and Enrichment Stages.
4. Included at the back of this handbook are brief solutions to each problem. These may be photocopied and given to students after 24 June. Do **not** give them to students earlier, even if they finish the Challenge earlier — friends in other schools may still be working on it. Do not use these solutions for marking the students' efforts, use the solutions and marking scheme in your Teacher Guide.
5. To help teachers with marking deadlines, it is suggested that students hand in their completed problems progressively.

B. The day when students are given their Challenge Problems

1. Arrange a meeting of all students participating in the Challenge.
 - a. Collect the information below for each student and record it on your spreadsheet, along with the name and mailing address of your school:
 - their last name and first name
 - school year, sex, date of birth
 - answers to the diversity information questions on the sheet.
 - b. Give the Student Problems books and any photocopied worksheets to each participant, making sure that they understand the 'Instructions to Students' at the front of their book and the 'Mark Allocation' at the back of their book.
 - c. Outline arrangements made for monitoring progress and handing in solutions.

C. The day when students hand in their attempts to the Challenge Problems

1. Collect the Teacher Guide.
2. Collect the students' Challenge scripts ensuring that they have clearly recorded their names on their scripts. For all students check and record whether the students have attempted the problems individually or have discussed them with a partner, and if so, have recorded their partner's name.

3. Marking Papers

- a. Award marks as shown in the mark allocation (see Teacher Guide). Marks are to be awarded in whole numbers only.

Be lenient — if you are unsure whether a mark should be awarded or not, award it (i.e. always give the student the benefit of the doubt). Be particularly *lenient* with reasoning marks, as students at this level are usually inexperienced at writing reasons. Students' solutions need not be as detailed as those given. Sometimes a brief interview with the student can clarify whether the student deserves the mark or not. *No half marks are to be awarded.*

- b. If an incomplete attempt is presented, which is different from the solution supplied, award marks for comparable work. If you are not sure, again give the student the benefit of the doubt. We have identified several alternative solutions for each problem.

4. Recording Results and Name Details

Submit results electronically on the spreadsheet that will be sent to your email, or contact mcya@amt.edu.au to request it.

To help us improve, please complete the electronic survey which we will send out in June.

D. Submit results to the Australian Maths Trust by 24 June

Return the results spreadsheet by email to:

mcya@amt.edu.au

Note: It is intended that the overall results, statistics and award certificates will be posted to participating schools in late July.

E. Follow-up Support for Students

1. It is suggested that teachers discuss the solutions of the Challenge problems with their students. A set of short solutions is provided at the back of this handbook, which may be photocopied, distributed to students and used as a basis for discussion.
2. To help further motivate and challenge the students, a number of extension problems are provided in the Teacher Guide. We recommend that students should be given these further problems to help reinforce the concepts and ideas already met in the Challenge problems and to provide further practice in problem solving.

3. Students in Years 5-10 who have done well or who have shown commitment in the Challenge should be encouraged to participate in the Maths Enrichment. It is not too late to enter as it is designed to be scheduled, by the school, in a convenient set of 12-16 weeks between April and September. For further information contact:

Australian Maths Trust

Tel: 02 6201 5136

Fax: 02 6201 5052

Email: mcy@amt.edu.au

4. **Australian Intermediate Mathematics Olympiad**

Students who do well or show commitment in the Maths Challenge and/or Enrichment and other talented students should be encouraged to enter the Australian Intermediate Mathematics Olympiad (AIMO) to be held on 8 September 2022. Registration is online and materials will be sent to the registered email address. It is a four-hour closed book contest, consisting of ten questions based on broad mathematical ideas. Students sit the AIMO at their own schools, and papers are sent to AMOC State Directors for marking.

The AIMO is the major identifier of students who will be invited to participate in the AMOC's other activities such as the maths extension programs and training schools.

How much may a Teacher help?

It is stated in the instructions to students that solutions to the problems must be completely the work of the student and that students may seek the use of resources such as textbooks or library books, computers and calculators, but may *not* seek help from other people. Where a problem is worked on with a partner (or partners), the final solutions must be written out by each entrant individually and in the student's own words.

However, where there are circumstances in which students cannot proceed, perhaps because of language or terminology, teachers may give some guidance. Further, general problem-solving techniques may be discussed.

It is suggested that the teacher negotiate arrangements whereby students report on their progress at regular intervals during the course of the Challenge. This will encourage the student to discuss any difficulties in getting started and to spread his or her efforts over the *four weeks*, rather than leaving it until the last minute.

The following guidelines are designed to assist teachers in deciding what they may tell students.

- Help if asked, but only give as much as will get the student thinking in the right direction.
- Give hints very sparingly, and never in such a way that a solution is directly revealed.
- Direct the student to consider a similar problem on a smaller scale.
- Suggest, where appropriate, that the student make a table, list, diagram, etc.
- Remind the student: 'There are more options than you think. Can you look at it another way?' Don't show the other way(s), but rather force the student to take a broader view and consider alternatives.
- Ask the student to give a *wrong* answer and explain why it is wrong. This may provide an entry point to a solution.
- Help clarify the student's thinking by asking questions such as:
 - Do you understand the question?
 - What are you told in the question?
 - What are you required to find?
 - Are you using all the information?
 - Do you understand the terminology?
 - What have you found so far?
- Make sure that the student has noted the need to show uniqueness of a solution to some problems. This means that full marks will not be awarded for a solution established by 'guess-and-check' or a spreadsheet, unless the student demonstrates that all possible values have been tried.

Teachers may assist with the terminology. For example, words such as 'adjacent' may need to be explained, as may the means by which scientific notation is displayed on the calculator. A general discussion of exponents and scientific notation may be necessary. Encourage students to look things up and lead them to suitable references and resources, if necessary. Don't ignore the potential of a mathematical dictionary.

The Instructions to Students in the Student Problem booklets advise that, except for specified student collaboration, students may not seek help from other people, including people whom they could contact via the internet. This has become a significant issue in recent years. Some students have submitted problems to one or more problem-solving websites and simply waited for a solution to be provided by another user. This form of plagiarism can usually be detected by the deviation in style, accuracy, and sophistication from the student's usual work.

The following suggestions are made regarding each question in the Challenge. *The teacher should help only when help is requested.*

MIDDLE PRIMARY PROBLEMS**MP1 Outer Sums**

- Suggest students experiment with triangles cut from cardboard.
- Advise students to read the introduction carefully to understand what arrangements are allowed.

MP2 Palindromes

- Ensure students know the meaning of 'sum', 'digit', and 'reverse'.
- Advise students that, in Part b, the order of the palindromes in a sum does not matter.

MP3 Quad Cuts

- Remind students of the number of sides and angles in the following: triangle (3), quadrilateral (4), pentagon (5), hexagon (6), heptagon (7).
- Reinforce the idea that the shapes named above do not need to be regular, that is, they do not need to have equal side lengths or equal angles.
- Clarify for students that a single straight dividing line goes all the way across the entire shape, even if it leaves the shape and then re-crosses it somewhere else.
- Explain the terms concave, convex, and reflex angle for students who may not have encountered them.
- Encourage students to draw with rulers their own convex and concave quadrilaterals for experimentation.

MP4 Near-square Rectangles

- Encourage students to make a set of NSRs from centimetre grid paper.
- Suggest students write down how many grid squares are in the NSRs up to NSR10.

UPPER PRIMARY PROBLEMS**UP1 Zipline Numbers**

- Remind students that 'descending order' is the same as 'decreasing order' and 'from highest to lowest', and is from left to right.
- Remind students that 0 is an even digit.
- Remind students that the smallest 5-digit number is 10000 and the largest is 99999.
- Notice that the customary space between the hundreds and thousands digits has been omitted in this problem.

UP2 Near-square Rectangles

- Encourage students to make a set of NSRs from centimetre grid paper.
- Remind students that the area of a rectangle (number of unit squares inside) is 'length \times width'.

UP3 Poly Cuts

- Remind students of the number of sides and angles in the following: triangle (3), quadrilateral (4), pentagon (5), hexagon (6), heptagon (7), octagon (8), nonagon (9).
- Reinforce the idea that the shapes named above do not need to be regular, that is, they do not need to have equal side lengths or equal angles.
- Clarify for students that a single straight line goes all the way across the entire shape, even if it leaves the shape and then re-enters it somewhere else as in the example in the introduction.
- Explain the terms concave, convex, and reflex angle for students who may not have encountered them.
- Encourage students to make their own convex and concave shapes for experimentation.

UP4 Mixed-up Birthdays

- Use a calendar and encourage students to cut out cards that they can physically move around.
- Calendars are available on the internet.

JUNIOR PROBLEMS**J1 Dropdown Numbers**

- Encourage students to use systematic counting.
- Observe that any arrangement of digits of a dropdown number gives another dropdown number unless it starts with 0.
- Remind students of the definitions of terms such as consecutive, average, and digit.

J2 Phancy Photos

- Ensure students understand the difference between portrait and landscape orientation.
- Suggest students place 12 mm × 16 mm pieces on a sheet of millimetre graph paper to help visualise the posters.

J3 Unit Triangles

- Encourage students to find a formula for the area, in unit triangles, of a parallelogram whose sides are along grid lines.
- Remind students that *grid points* are the intersections of grid lines.
- Provide students with isometric (triangular) grid paper. Note that most printed isometric grid paper will need to be rotated so that grid lines align with the given diagrams.

J4 GoGo

- Encourage students to be systematic in their approach to this problem.
- Suggest students use copies of grids found online and photocopies of the patterns in this problem to explore ideas.

J5 Towers and Cities

- Suggest students list the values of some single towers.
- Advise students to work systematically to avoid missing cases.

J6 Mixed-up Birthdays

- Encourage students to use a calendar and cut out cards that they can physically move around.
- Calendars are available on the internet.
- Discuss what a leap year is and what that means.

INTERMEDIATE PROBLEMS

I1 DPS Chains

- Encourage students to experiment with making their own DPS chains so they become familiar with the process and how it terminates.
- Encourage students to work systematically when constructing DPS chains backwards.

I2 Triangle Folding

- Suggest students make a large paper right-angled isosceles triangle that they can fold and use to explore the problem.
- Encourage students to keep their answers exact as simplified surds.

I3 GoGo

- Encourage students to be systematic in their approach to this problem.
- Suggest students use copies of grids found online and photocopies of the patterns in this problem to explore ideas.

I4 Slot Cars

- Remind students that a formula for the circumference of a circle is $C = 2\pi r$.
- Encourage students to use grid paper or experiment with square cut-outs representing the two types of plate.
- Remind students that not every square in the 4×4 grid has to be used.

I5 Stick or Roll

- Encourage students to think about probabilities as the proportion of ways in which events may occur.
- Tables or tree diagrams can be used to keep track of the proportions of ways (probability) that multistage events occur.

I6 Tandem Cycling

- Suggest students draw diagrams to support their reasoning. These could show the changeover points along the way home, and who is on the tandem and its direction between these points.
- Encourage students to work with fractions to avoid rounding errors.
- Decimal *answers* are acceptable.

Some Statistics from 2021

MIDDLE PRIMARY

The cut-offs for certificates in Middle Primary were as follows:
(Each question is marked out of 4, so there were 16 marks available.)

Award	Score	Distribution of Awards
Distinction	16-14	10%
Credit	13-10	34%
Proficiency	9-6	39%

The remaining students were awarded a Certificate of Participation.

UPPER PRIMARY

The cut-offs for certificates in Upper Primary were as follows:
(Each question is marked out of 4, so there were 16 marks available.)

Award	Score	Distribution of Awards
Distinction	16-14	19%
Credit	13-10	34%
Proficiency	9-6	30%

The remaining students were awarded a Certificate of Participation.

JUNIOR

The cut-offs for certificates in the Junior Level were as follows:
(Each question is marked out of 4, so there were 24 marks available.)

Award	Score	Distribution of Awards
High Distinction	24-22	11%
Distinction	21-19	18%
Credit	18-14	21%

The remaining students were awarded a Certificate of Participation.

INTERMEDIATE

The cut-offs for certificates in the Intermediate Level were as follows:
(Each question is marked out of 4, so there were 24 marks available.)

Award	Score	Distribution of Awards
High Distinction	24-21	14%
Distinction	20-18	18%
Credit	17-13	24%

The remaining students were awarded a Certificate of Participation.

2022 DIRECTOR'S HANDBOOK