AUSTRALIAN MATHS TRUST | MATHS ENRICHMENT

Important changes to the Euler stage of Maths Enrichment from 2024

Overview

The *Euler Student Notes* has undergone a substantial review and a new edition will be published for the 2024 Maths Enrichment program. Changes include:

- new content
- new topics not previously covered
- improved alignment with other stages in the program
- larger booklet and font size.

From 2024, the *Euler Student Problems* will contain 10 problems rather than 12 as previously.

Electronic copies will be available for participating schools prior to the mailout of materials.

All students and teachers using Enrichment materials should transition to the new edition of the *Euler Student Notes* from 2024 as the previous version will not align with the 2024 program materials.

Details

A. Program changes

The new edition of the Euler Student Notes contains 10 chapters (see Section C for details). From 2024, the accompanying Student Problems will include one problem for each chapter. So there will 10 problems in total, compared to 12 in previous versions. Problems will still be worth 4 marks each for a total of 40 marks (down from 48 marks).

While most topics in the previous edition of the Student Notes will still be relevant, students will need to use the new edition to support their progress through the 2024 Student Problems.

In several cases, content from two or more chapters of the previous edition has been consolidated into a single chapter (see Section D for details). While the number of chapters has reduced, there is an increase in the number of different topics explicitly covered. The overall number of exercises has decreased somewhat (by around 10%). On balance, the workload for students should be comparable.

We hope you and your students enjoy using these new materials. If you have any queries or comments about these changes or the program more broadly, contact competitions@amt.edu.au.

B. General comments about the new student notes

Considerable thought has been given to this review and to making Euler Student Notes more accessible, particularly for students undertaking the work with minimal support from a specialist teacher. In places, this involves more detailed introduction and discussion of new ideas. More experienced students, particularly those with exposure to the Newton and Dirichlet stages, may be able to skim through some sections. Similar comments apply to topics that have considerable overlap with the Australian Curriculum. A detailed list of topics in the new edition is in Section C.

C. CHAPTERS IN THE NEW EDITION (from 2024)

This table outlines the new chapter titles, links to preceding *Enrichment stages*, numbers of pages and exercises, section titles and brief descriptions of the topics. Chapters 3, 5 and 7 (marked *) contain content that is entirely new to the *Euler* stage, although they build on content in earlier stages.

1	**Primes and the Euclidean algorithm** (first 3 sections recap *Dirichlet* Chapter 6, some overlap with Australian Curriculum)		11 pages 20 exercises
	Prime decomposition	Terminology, prime factorisation, factor trees, index form	
	Least common multiple	LCM via listing multiples, LCM via prime decompositions	
	Greatest common divisor	GCD (HCF) via listing factors, GCD via prime decompositions	
	We have nothing in common …	Relatively prime/coprime numbers	
	The Euclidean algorithm	GCD (HCF) via repeated subtraction/repeated division	
	Primes, lcm and gcd in problem solving	Mixed problems involving decompositions, LCM and GCD (HCF)	
2	**Advanced counting** (first 3 sections recap *Dirichlet* Chapter 5)		11 pages 18 exercises
	When should I add and when should I multiply?	Mutually exclusive events, simple addition principle, independent events, multiplication principle, problems with and without repetition	
	When should I subtract?	Generalised addition principle (union/intersection), complements	
	What if order matters?	Factorials, $n!$ notation, groupings, permutations, ${}^{n}P_{r}$ notation	
	What if order doesn't matter?	Combinations, ${}^{n}C_{r}$ notation, sum of combinations = 2^{n}	
3	**Graphs and networks*** (builds on graph colouring in *Newton* Chapter 6)		11 pages 18 exercises
	Introduction to graphs	Introductory concepts, terminology and exploration	
	Some special graphs	Polygons, polyhedra, stars, complete graph	
	The handshaking lemma	Edge counting via degrees, non-exsitence proofs	
	Planar graphs and Euler's formula	Face counting, infinite region, Euler's formula $V-E+F=2$	
4	**Methods of proof** (first general introduction to proof techniques)		6 pages 14 exercises
	Direct proof and contradiction	Algebra-based introduction to proof techniques	
	The pigeonhole principle	PHP, generalised PHP	
5	**Introduction to Diophantine equations*** (relies on familiarity with algebra and 2-step equations)		6 pages 14 exercises
	Diophantus of Alexandria	Introduction to problems with integer solutions	
	Equivalent fractions	Problems reducing to search for equivalent fractions, simple digit problems	

	Linear Diophantine equations	Solution of ax + by = c via repeated subtraction or addition, divisibility property shortcuts, more complex digit problems
	The 'plus zero' trick	Using one solution to find others, general solution
6	**Chase that angle** (large overlap with Australian Curriculum)	14 pages 30 exercises
	The basics	Introduction, terminology
	Naming conventions	Naming points, lines (etc.), angles, polygons
	What is a right angle exactly?	Definitions: right, straight, acute, obtuse, reflex
	Using algebra	Solving equations for unknown angles, simple proofs using relationships between variables
	Lines that cross and lines that don't	Vertically opposite, parallel line properties, angle sum of triangle proofs
	Adding construction lines	Mixed problems
	More about angle sums	Exterior angles of triangles
	A special triangle	Isosceles triangles
	Even more about angle sums	Angle sum and exterior angles of other polygons
7	**Number bases*** (first 3 sections recap and extend *Newton* Chapter 8)	6 pages 14 exercises
	Decimal recap	Introduction and place-value recap
	Other bases	Converting to and from decimal (base 10)
	Arithmetic	Addition, subtraction, multiplication and division in other bases
	Problem solving with bases	Finding unknown bases, finding unknown numbers
8	**Sequences and figurate numbers** (extends sequence introduction in *Newton* Chapter 5 and *Dirichlet* Chapter 2)	11 pages 15 exercises
	A clever shortcut	Introduction and motivation
	Arithmetic sequences	Terminology, notation, $t_n = a + (n–1)d$
	Arithmetic series	Terminology, notation, $S_n = n/2(a + l) = n/2\ (2a + (n - 1)d)$
	Figurate numbers	Triangular, square and pentagonal numbers, deriving algebraic formulas, relationships via diagram dissections and/or algebra
9	**Modular arithmetic** (first 4 sections recap and extend *Dirichlet* Chapter 8)	7 pages 14 exercises
	From 'base' to 'modulo'	Introduction and motivation
	Remainders and equivalence	Definitions and examples
	Basic arithmetic	Calculating remainders using addition, subtraction, multiplication
	What about division?	Modular inverses, solving equivalences
	Powers	Shortcuts for calculating remainders of large powers
10	**Mixed problem solving** (replaces 'Problems I like' chapters)	3 pages 18 exercises
		Problems that require combining ideas from two or more chapters, emphasis on proofs, includes table of topic hints

D. CHAPTERS FROM THE PREVIOUS EDITION (prior to 2024)

This table outlines how content from the previous edition has been repurposed in the new edition.

1, 8, 12 Problems I like/enjoy sharing

Where there are direct links to topics or techniques explicitly covered in the new edition, some of these problems have been repurposed and extended in the relevant chapter or in the new Chapter 10, Mixed *problem solving.* Unused problems will be considered for the next edition of the *Gauss Student Notes*.

2, 3, 4 Primes and composites, LCM, HCF and the Euclidean algorithm

These chapters have been consolidated into the new Chapter 1, *Primes and the Euclidean algorithm.* Counting factors via prime decompositions has moved to *Dirichlet* and will be revisited in the new edition of Gauss. Content relating HCF and the graphs of linear relations has been omitted.

5, 6 Arithmetic sequences, Figurate numbers

These chapters have been consolidated into the new Chapter 8, *Sequences* and figurate numbers.

7 Congruences

This chapter has been adapted to better align with *Dirichlet* Chapter 8, *Clock arithmetic*, and appears as the new Chapter 9, *Modular arithmetic*.

9 Find that angle

Much of the content in this chapter remains intact in the new Chapter 6, *Chase that angle*. Introductory sections have been considerably expanded to support students with limited prior exposure to geometry terminology and notation.

10 Counting Techniques

This chapter forms the basis of *Dirichlet* Chapter 8, *Counting techniques*. The new Chapter 2, *Advanced counting*, builds on and extends this content, but circular arrangements have been omitted.

11 The Pigeonhole Principle

This topic now appears in the new Chapter 4, *Methods of Proof*. An introductory section on direct proof and proof by contradiction has been added.