SELECTED INFORMATION FROM THE ENRICHMENT HANDBOOK

(each school gets at least one copy of the Handbook each year. It contains further information specific to that year.)

ABOUT THE ENRICHMENT STAGE

The seven stages are independent and none is a pre-requisite for any other. They are Ramanujan, Newton, Dirichlet, Euler, Gauss, Noether and Pólya and all include Student Notes, a Student Problems book and Teacher Guide.

Encourage students to work through the material of each chapter of the Student Notes attempting solutions to the exercises as they go, initially without recourse to the solutions provided in the Notes. Students can check and compare their solutions with the given solution, or, if a solution is not found, use the solution to learn techniques that may be able to be applied to another problem.

We hope that as students work through this material they will add to their set of mathematical tools and improve their problem-solving skills. Many of the exercises require a non-standard use of the tools while some simply reinforce a concept which has been introduced. The students are encouraged to look for 'neat' solutions rather than achieve the result through a lengthy or awkward approach. At the same time, persistence is often required to solve the problems and is a quality to be fostered in all young problem-solvers.

Ramanujan
This is designed for mathematically able Year 4 and 5 students.
The topics covered are:

- Estimation
- Special numbers
- Counting techniques
- Colouring problems
- Fractions
- Clock arithmetic
- Ratio
- some problem solving techniques

The Student Problems book has eight problems, each one covering the work given in a particular chapter of the Student Notes.

Newton
This has eight chapters and is designed for mathematically able Year 5 and 6 students. It is also most appropriate for use with Years 7 and 8 students. The topics covered are:

- Polyominoes
- Polyhedra
- Patterns
- Problem solving
- Fast arithmetic
- Pre-algebra concepts
- Divisibility

The Student Problems book has eight problems, each one covering the work given in a

particular chapter of the Student Notes.

Dirichlet

This is designed for students in Years 6 or 7. Three of the eight chapters revolve around a story which illustrates some problem-solving techniques: using logic, solving a simpler problem, and working backwards. The other five chapters cover:

- Tessellations
- Time, distance, speed
- Recurring decimals
- One-handed arithmetic
- Working with patterns

Almost all of the material presented is mathematics not usually taught in school at any level and so is appropriate for enrichment purposes.

Euler

This is designed for interested and talented students in Years 7 and 8. The topics considered in this series are:

- Primes and composites
- Highest common factors
- Figurate numbers
- Properties of angles
- The Pigeonhole Principle
- Least common multiples
- Arithmetic sequences
- Congruences
- Counting techniques

Chapters 1-4 involve very little algebra. Direction to a suitable textbook may be required to assist with algebra in subsequent chapters.

Gauss

This is designed for talented students in Years 8 and 9. It introduces the use of computer spreadsheets such as Lotus 1-2-3 and Excel. The topics covered include:

- Parallels
- Pythagoras' Theorem
- Diophantine equations
- Congruence
- Similarity
- Spreadsheets
- Counting techniques

Each chapter introduces new mathematics and presents some key mathematical ideas followed by some illustrative examples with suggested approaches and sets of exercises for which there are fully worked solutions.

Noether

This is designed for talented students in Years 9 and 10, following on from Gauss. The topics considered are:

- Expansion and factorisation
- Number bases
- Methods of proof
- Sequences and series
- Inequalities
- Congruence

- Circles
- Tangents

Spreadsheets may also be useful for some problems.

Pólya

This is specifically designed for the top 5–10% of Year 10 students. It contains a supplement on Euclidean geometry. The first four chapters offer an introduction to some selected topics in algebra. The last six chapters are a commentary on the use of the supplement which contains a selection of geometric topics. The chapters are:

- Expansion and factorisation
- Polynomials
- Series
- Inequalities
- Congruent triangles
- Parallelism
- Areas of related figures
- Chord and angle properties
- Concyclic points
- Tangency

It would be useful for students to compile an ongoing summary of the facts and techniques learned and use them to obtain their own solutions to the examples and exercises.

Instructions and Information for Students

Unlike the student problems in the MCYA Challenge Stage, the problems in the Enrichment Stage are generally based on topics studied in the Student Notes, whether in Ramanujan, Newton, Dirichlet, Euler, Gauss, Noether or Pólya. In the Student Notes there are a number of examples and exercises with solutions for most topics so that students have an opportunity to study similar questions to the given problems.

Make sure that the students understand the instructions given inside the Student Problems books. These are as follows:

1. Before attempting the problems in this booklet, work through the appropriate sections of the Student Notes.

2. Solutions to the problems must be completely your own work. You may use resources such as text books or library books but may not seek help from other people including those you could contact via the internet. A calculator or a computer may be used,
   but be sure that any programs used are fully and carefully explained.

3. Submit careful solutions to these problems according to the schedule provided by your school's MCYA Enrichment Stage Director.

4. Each problem will be scored out of four marks. Marks will be awarded for clearly expressed arguments and careful reasoning. It is important to submit partial solutions
   to problems you have not completed as these may be worth some marks.

Further helpful information for students

- The chapters are independent of each other and so can be read in any order.

- While there will be some problems where the only technique available is the 'worry-it-to-death' approach—testing an exhaustive (in both senses of the word!) number of cases—be on the lookout for neat short solutions. This will require the use of the mathematical ideas presented and some ingenuity.

- Additional help can be requested by contacting your teacher or school's MCYA Director for suitable resources or texts.

- A schedule for planning your chapter study and for submission of problems will be provided by your school's MCYA Director.

- Some problems may prove to be frustrating or too difficult. It is not necessary to submit all the problems; leave out some and do the later ones.

- We recommend that students compile an ongoing summary of the facts and techniques learnt in Enrichment. This will help with further studies in mathematics.

- The Student Notes will be useful in future secondary school and tertiary studies in mathematics and will be a valuable addition to your library.

HOW MUCH MAY A TEACHER HELP?

As stated in the instructions to students, solutions to the problems must be completely their own work. However if there are circumstances when students cannot proceed, perhaps because of language or terminology, then teachers may give some guidance. Further general problem-solving strategies may also be discussed.

The following general guidelines are designed to assist teachers in deciding what they may tell students.

- Emphasise the importance of reading the relevant parts of the text and solving the related problems.

- Help if asked, but only give as much as will get the student thinking in the right direction.

- Give hints very sparingly and never in such a way that a solution is directly revealed.

- Give general problem-solving hints including:
  - Direct the student to consider a similar problem on a different scale.
  - Suggest where appropriate that the student make a table, list, diagram etc.
  - Remind the student 'There are more options than you think. Can you look at it another way?' Don't show the other way(s) but rather help them to take a broader view and consider alternatives.

- Help the student to clarify their thinking by asking questions such as:
  - Do you understand the question?
  - What are you told in the question?
  - What are you required to find?

- Are you using all the information?

- Do you understand the terminology?

- What have you found so far?

- Can you simplify the problem?

- How about writing something down?

- Assist with the terminology, particularly with students from Years 5 to 8, and with students whose first language is not English. For example, words such as 'random' may need to be explained as may the means by which scientific notation is displayed on the calculator. A general discussion of exponents and scientific notation may be necessary.

- Encourage students to look for meanings of words and lead them to suitable references and resources in the library if necessary. Don't ignore the potential of a mathematical dictionary.

- Emphasize that marks will be awarded for reasoning and that arguments need to be written out fully and carefully.

SCHEDULES AND MARKING

STUDENT SCHEDULES AND REGULAR REPORTING

Consider the time available in your school from the commencement (April-June) of the MCYA Enrichment Stage through to its finish in late September. There will be school examinations, holidays, excursions and so on. There are eight problems in Ramanujan, Newton and Dirichlet, twelve problems in Euler and Gauss, and sixteen problems in Noether and Pólya that the students will have to attempt during this period. Thus realistically divide the time available to the student (and yourself) into a reasonable schedule. It is not necessary to use the entire six months.

In designing the schedule, organise the deadlines for submission of Enrichment Stage problems so that you can correct them quickly. For example, you may wish to set aside two free periods on say Tuesday afternoon for marking the problems, so make the deadline 9 am Tuesday for the students.

Note that such a schedule will mean that some students complete all the exercises in the Student Notes and then attempt the Enrichment Stage problems, while other students will only do a few exercises before attempting the Enrichment Stage problems.

We recommend that you organise regular classes once or twice a week in order to discuss topics and exercises in the Student Notes and, later, the students' submitted solutions to the Enrichment Stage problems.

MARKING AND RECORDING RESULTS

We suggest that you mark papers and record results on a regular basis.

A results spreadsheet and School Details form/MCYA Questionnaire will be emailed to your MCYA Enrichment Stage Director towards the end of the timeline. Contact the AMT office at mcya@amt.edu.au if you require these documents earlier.

1. Award marks as shown in the marking scheme.

2. If an incomplete attempt is presented which is different from the solution supplied, award marks for comparable work. To assist you to do this we have identified several alternative solutions for many problems and summarized the work for which each mark is awarded. If you are not sure what mark to award, give the student the benefit of the doubt. Awarding a 0 will ensure the student will receive a certificate.

3. A Participation certificate will not be awarded to a student if all questions for that student are marked 'NOT ATTEMPTED'.

DISCUSS CORRECTED WORK ON A REGULAR BASIS

It is important to discuss the solutions with the students as soon as possible after the questions have been attempted by all participating students and marked. To help teachers to further challenge and motivate their students, extra sample problems and their solutions are available on the AMT website www.amt.canberra.edu.au/wuenr.html.

4. On completion of marking, record each student's score in the appropriate place on the results spreadsheet or on the back of their answer sheet making sure that the code for Ramanujan, Newton, Dirichlet, Euler, Gauss, Noether or Pólya has been marked.

5. Give out the solutions to the relevant problems and discuss the scores with the students.

6. Please indicate on the School Details form/MCYA Questionnaire any comments, suggestions, guidance etc. about the Enrichment Stage program. We need your help in order to improve procedures and concepts of this important initiative of the Australian Mathematical Olympiad Committee.

7. Complete the Enrichment Stage results spreadsheet and electronic School Details form/MCYA Questionnaire return by email to mcya@amt.edu.au or return the completed answer sheets and School Details form/MCYA Questionnaire in the return address envelope.

8. Schools can expect to receive results and certificates for the Enrichment Stage by mid-November.

STUDENT SUCCESS

For some students the MCYA Enrichment Stage may be the first time they have encountered mathematical ideas they do not immediately understand. Explain that not all students are expected to complete all the work in the scheduled time, and not to be too concerned if they cannot meet their expectations. The most important thing is that students understand clearly all the work they have studied, and extend their mathematical knowledge as far as possible.

It is expected that some of the topics in the various books of Ramanujan, Newton, Dirichlet, Euler, Gauss, Noether and Pólya may not have been completed by many students. Why not encourage them to work on the material during the last couple of months of the year?

MCYA Enrichment Stage Package

1. The MCYA Enrichment Stage package includes the following:

(a) Director's materials

- Letter to the school's MCYA Enrichment Stage Director
- MCYA Enrichment Stage Director's Handbook
- Answer sheets
- School Details form/MCYA Questionnaire (yellow)
- Return address envelope (plastic) and cardboard protector
- AMOC High School Mathematics Problem-Solving Program brochure
- Australian Mathematics Trust Publications order form

(b) Enrichment Stage materials

Each student registered will receive

- 1 x Student Notes
- 1 x Student Problems

The MCYA Enrichment Stage Director will receive a Teacher Set/s (for every stage in which students are participating) containing

- 1 x Student Notes
- 1 x Student Problems
- 1 x Teacher Set for 20 entries or fewer
  (1 extra Teacher Set for 21-40 entries)
  (1 extra Teacher Set per multiple of 20 entries)